Homework
due on Monday, November 25

Read carefully Chapter 5 and Chapter 7, sections 37, 39 of Hartshorne's book. Solve the following problems:

Problem 1. Consider an isosceles triangle with base of length 18 and height of length 16. Divide this triangle into several polygonal pieces from which a square of side 12 can be assembled (use 1 cm as a unit). Explain your solution carefully and provide the actual pieces made out of a thin cardboard (or paper).

Problem 2. Consider 4 circles S_1, S_2, S_3, S_4. Suppose that S_1, S_2 intersect at A_1 and A_2, circles S_2, S_3 intersect at B_1 and B_2, circles S_3, S_4 intersect at C_1 and C_2, and circles S_4, S_1 intersect at D_1 and D_2. Suppose furthermore that the points A_1, B_1, C_1, D_1 are on a circle S (or on a line). Prove that the points A_2, B_2, C_2, D_2 are also on a circle (or on a line).

Hint: Consider a circular inversion with center A_1 (in any circle with center A_1). What will hapen to S_1, S_2 and S? What will happen ntto the other circles? You should be able to apply our old theorem about Miquel point.

Problem 3. No three of the points A, B, C, D are collinear. Prove that the angle between the circumcircles of triangles ABC and ABD is the same as the angle between the circumcircles of triangles ACD and BCD.

Hint: Perform a circular inversion in a circle with center A. What happens to the angles in question?

Problem 37.3

Problem 37.14

Problem 37.18

Problem 24.16. Hint: Let b be the longest side of the triangle. Use the method from class to dissect the triangle into 3 pieces and assemble them into a rectangle with one side b. Then follow the proof of Proposition 24.8 (explain why it can be used).

Problem 24.17 Hint: compare the longest side of the triangle to the diameter of the square.