Homework 1

Problem 1: Prove that the product of three consecutive natural numbers is never a square of an integer. Prove more generally, that it cannot be an m-th power of an integer for any $m > 1$.

Problem 2: Let k be a positive integer. Prove that the numbers $4k + 1$, $6k + 1$ and $9k + 1$ cannot be all squares.

Problem 3: Prove that an integer of the form $5k \pm 2$ cannot be a square.

Problem 4: A function $f : [0, 1] \longrightarrow \mathbb{R}$ satisfies:

1. $f(0) = 0$;
2. $f(1 - x) = 1 - f(x)$ for all x;
3. $f(x/5) = f(x)/3$ for all x.

Compute $f(26/31)$.

Problem 5: A non-decreasing function $f : [0, 1] \longrightarrow \mathbb{R}$ satisfies:

1. $f(0) = 0$;
2. $f(1 - x) = 1 - f(x)$ for all x;
3. $f(x/5) = f(x)/2$ for all x.

Compute $f(16/2005)$.