Homework 6

Problem 1: Problem 4.3.9

Problem 2: Express the generating function of the sequence

$$a_0 = 2,\ a_1 = 5,\ a_n = 5a_{n-1} - 6a_{n-2}$$

as a fraction of 2 polynomials. Use partial fractions to get an explicit formula for a_n.

Problem 3: Prove that

$$\sqrt{2(a^2+b^2)}+\sqrt{2(b^2+c^2)}+\sqrt{2(c^2+a^2)} \geq \sqrt{3(a+b)^2+3(b+c)^2+3(c+a)^2}$$

for any real numbers a, b, c.

Problem 4: a) Prove that

$$\frac{x_1^u + x_2^u + ... + x_n^u}{n} \leq \left(\frac{x_1 + x_2 + ... + x_n}{n}\right)^u$$

for any positive real numbers $x_1, ..., x_n$ and any $u < 1$.

b) Prove that

$$\frac{\sqrt{x_1}+\sqrt{x_2}+...+\sqrt{x_n}}{\sqrt{n-1}} \leq \frac{x_1}{\sqrt{1-x_1}} + \frac{x_2}{\sqrt{1-x_2}} + ... + \frac{x_n}{\sqrt{1-x_n}}$$

for any positive real numbers $x_1, ..., x_n$ such that $x_1 + ... + x_n = 1$. Hint: Consider the function $f(x) = x/\sqrt{1-x}$.

Problem 5: Problem 5.5.34.

Problem 6: Problem 5.5.32.

Problem 7: Problem 5.5.39.

Problem 8: Problem 5.5.43.

Problem 9: Problem 5.5.47.