Homework 5

Problem 1: Compute $\int_0^2 \frac{x^3 dx}{6x^2 - 12x + 8}$. Hint: what is $(x - 2)^3$?

Problem 2: Two rivers run parallel 2 miles appart. Two cities A and B lie between the rivers; each city is equidistant from the rivers and the cities are 3 miles appart. A scientist wishes to travel from A to B, collecting a sample of water from each river during his journey. What is the length of the shortest path he can follow. Justify your answer.

Problem 3: There are 2000 points on a circle and each point is given a number which is equal to the avarage of the numbers of its two nearest neighbours. Show that all the numbers must be equal.

Problem 4: Let $a_1, a_2, ..., a_n$ be positive integers all of whose prime divisors are ≤ 13 .

- Show that if $n \ge 65$ then there exist two of these integers whose product is a perfect square.
- Show that if $n \ge 193$ then there exist four of these integers whose product is a perfect fourth power.

Problem 5: There are 2n people attending a meeting. Each person knows at least n other participants. Show that it is possible to accomodate the participants in n rooms so that each room is occupied by two participants who know each other.