Homework 6

due on Monday, November 20

Problem 1. Initially, we are given the sequence $1, 2, ...100$. Every minute, we erase any two numbers u and v and replace them with the value $uv + u + v$. Clearly, we will be left with just one number after 99 minutes. What is this number? Justify your answer.

Problem 2. Start with the set $\{3, 4, 12\}$. You are then allowed to replace any two numbers a and b with the new pair $0.6a - 0.8b$ and $0.8a + 0.6b$. Can you transform the set into $\{4, 6, 12\}$?

Problem 3. Show that if every room in a house has an even number of doors, then the number of outside entrance doors must be even as well.

Problem 4. Seventeen people are at a party. It turns out that for each pair of people present, exactly one of the following statements is always true: "They haven't met", "They are good friends", or "They hate each other". Prove that that must me a trio (3) of people, all of whom are either mutual strangers, mutual good friends, or mutual enemies.

Problem 5. A large house contains a television set in each room that has an odd number of doors. There is only one entrance to this house. Show that it is always possible to enter the house and get to a room with a television set.

Problem 6. A domino consists of two squares, each of which is marked with $0, 1, 2, 3, 4, 5$ or 6 dots. Verify that there are 28 different dominos. Is it possible to arrange them all in a circle so that the adjacent halves of neighboring dominos show the same numbers?