Exam 1

November 20, 2006

In order to get full credit a correct solution must be written carefully, with detailed explanation of all steps. Each problem is worth 10 points.

Problem 1. Let S be a set consisting of 10 natural numbers not exceeding 100. Show that there are two different subsets of S such that the sums of the numbers in each subset are equal. (Hint. Recall that a set with n elements has $2^n - 1$ non-empty subsets)

Problem 2. Numbers $1, 2, 3, ..., 2006$ are written on a blackboard. Every now and then somebody picks two numbers a and b and replaces them by $a - 1$, $b + 3$. Is it possible that at some point all numbers on the blackboard are even? Can they all be odd?

Problem 3. Prove that there is no integer $n > 2$ such that $n(n + 6)$ is a square of an integer.

Problem 4. Find

$$\int_{-1}^{1} \frac{x^{2006} dx}{2^x + 1}.$$