Exam 1, Math 488A

Friday, November 30

Problem 1. Compute the integerals

$$\int_0^\pi \frac{x^2 \sin x}{x^2 + (\pi - x)^2} dx.$$

$$\int_0^\pi \frac{x^3 \sin x}{3x^2 - 3\pi x + \pi^2} dx.$$

Problem 2. A set of 10 different integers is selected from $\{1, 2, 3, ..., 18\}$. Prove that among the selected integers there are two numbers which differ by 3.

Problem 3. Positive numbers a, b, c satisfy $a^{-1} + b^{-1} + c^{-1} = 3$.

a) Prove that $abc \geq 1$

b) Prove that $(a + b)(a + c)(b + c) \geq 8$. When does the equality hold?

Problem 4. Numbers $1, 2, ..., 2007$ are written on a blackboard. Every now and then somebody picks two numbers a and b and replaces them by $a - 3$, $b + 1$. Is it possible that at some point all the numbers on the blackboard are odd? Justify your answer.