Homework

due on Friday, September 7

Problem 1. N seemingly identical coins are given. It is known however that one of the coins is fake and it is heavier than all the other coins (which are genuine). Find a procedure to determine the fake coin using smallest possible number of weighings on a balanced scale.

Problem 2. Problem 1.3.16 from the textbook.

Problem 3. Three scholars are seated in a row one behind the other. They are shown three red hats and two blue hats. Each is blindfolded, a hat is placed on each head, and the blindfolds are removed. Starting from the back, each scholar is then asked what color hat was on his or her head, and to explain how they knew for certain the hat was that color.

The scholar in the back, who could see two hats, could not answer. The scholar in the middle, who could see one hat, could not answer either. The scholar in the front, who could see no ones hat, was able to answer correctly, and explain how she knew the color for certain.

What color hat did the scholar in front have on her head, and how was she able to answer correctly?

Problem 4. Here is a general problem:

You and N others are sitting in a circle, so that everyone can see everyone else. Right now, you all have all the time you need to discuss strategy.

When everyone is ready, you will all be blindfolded. While you are blindfolded, a hat will be placed on your heads. Each hat is one of $N + 1$ colors, but the colors may be duplicated.

Everyone's blindfold is then removed. Everyone can see the color of everyone else's hat, but no one can see their own, and no one can communicate in any way with anyone else. Everyone writes, on a piece of paper that no one else can see, a guess at the color of their hat. If everyone is wrong, everyone loses, but if at least one person guesses correctly, everybody wins something nice.

Does any strategy guarantee victory? If yes, what is it? If not, why not?

Try to solve it for $N = 1$ and $N = 2$.