Homework

due on Friday, September 21

Problem 1. A sequence $a_1, a_2, ...$ of positive real numbers satisfies $a_n \leq a_{2n} + a_{2n+1}$ for every n. Show that the series $a_1 + a_2 + a_3 + ...$ diverges.

Problem 2. Find all integrs $m \neq n$ such that $m^n = n^m$.

Problem 3. Let a, b be positive integers. Prove that

$$|\frac{a}{b} - \frac{1}{\sqrt{2}}| > \frac{1}{4b^2}.$$

Problem 4. Let n be a positive integer divisible by 24. Let s be the sum of all positive divisors of $n - 1$. Prove that s is divisible by 24.