Homework

due on Friday, October 5

Problem 1. Prove that among any five points selected inside an equilateral triangle with side equal to 2, there always exists a pair at the distance not greater than 1.

Problem 2. 21 integers are selected from $\{1, 2, 3, ..., 400\}$. Prove that two of them, say x and y, satisfy $0 < |\sqrt{x} - \sqrt{y}| < 1$.

Problem 3. Nine distinct points with all coordinates integral are selected in the space. Prove that the line segment with ends at certain two of these points contains in its interior a point with all coordinates integral.

Problem 4. 2004 flies are inside a cube of side 1. Show that some 3 of them are within a sphere of radius 1/11.

Problem 5. Six points are positioned in a 3×4 rectangle. Prove that there are two of them at the distance not exceeding $\sqrt{5}$ from each other.

Problem 6. Given a set $A \subset \{1, 2, ..., 100\}$ of ten integers. Prove that it is possible to select two disjoint non-empty subsets S, T of A whose members have the same sum. **Hint.** How big can the sum of leass than 10 elements from A be? Solve the problem with the assumption "disjoint" replaced by "different" first. Recall that a set with n elements has 2^n subsets.