Homework

due on Friday, October 26

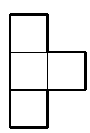
Problem 1. Initially, we are given the sequence $1, 2, ...100$. Every minute, we erase any two numbers u and v and replace them with the value $uv + u + v$. Clearly, we will be left with just one number after 99 minutes. What is this number? Justify your answer.

Problem 2. Start with the set $\{3, 4, 12\}$. You are then allowed to replace any two numbers a and b with the new pair $0.6a - 0.8b$ and $0.8a + 0.6b$. Can you transform the set into $\{4, 6, 12\}$?

Problem 3. Show that if every room in a house has an even number of doors, then the number of outside entrance doors must be even as well.

Problem 4. A large house contains a television set in each room that has an odd number of doors. There is only one entrance to this house. Show that it is always possible to enter the house and get to a room with a television set.

Problem 5. Is it possible to cover a 10×10 square with 25 $T-$tetrominoes? (a $T-$tetromino is a plane figure shown below, constructed from four unit squares arranged in the form of T).



Is it possible to cover a 10×10 square with 25 $L-$tetrominoes? (an $L-$tetromino is a plane figure shown below, constructed from four unit squares arranged in the form of L)