Homework

due on Friday, November 9

Problem 1. There are 2n people attending a meeting. Each person knows at least n other participants. Show that it is possible to accommodate the participants in n rooms so that each room is occupied by two participants who know each other.

Problem 2. Find smallest number $f(n)$ for which the following statement is true: any graph with $2n+1$ vertices and $f(n)$ edges contains three vertices such that any two of them are joined by an edge (i.e. it contains circuit of length 3). Remark: In class we showed that for graphs with even number $2n$ of vertices the analogous number is n^2+1.

Problem 3. In some society any two members who know each other do not have any other common acquaintances. Any two members who do not know each other have exactly two common acquaintances. Prove that each member knows the same number of other members.

Problem 4. Each of the three companies employs n people. Each employee knows exactly $n+1$ employees from other two companies. Prove that one can choose one employee from each company so that the three know each other.

Problem 5. 10 people attend a party. Among any three of these people there are two which know each other. Prove that there are four people at the party who all know each other. Is the same true when 10 is replaced by 9?