Homework

due on Wednesday, November 28

Problem 1. Prove that

$$\frac{\sqrt{x_1}+\sqrt{x_2}+...+\sqrt{x_n}}{\sqrt{n-1}} \leq \frac{x_1}{\sqrt{1-x_1}}+\frac{x_2}{\sqrt{1-x_2}}+...+\frac{x_n}{\sqrt{1-x_n}}$$

for any positive real numbers $x_1, ..., x_n$ such that $x_1 + ... + x_n = 1$. Hint: Consider the function $f(x) = x/\sqrt{1-x}$.

Problem 2. Let a_1, a_2, a_3, a_4, a_5 be positive real numbers such that $a_1 + a_2 + a_3 + a_4 + a_5 = 1$. Prove that

$$(\frac{1}{a_1}-1)(\frac{1}{a_2}-1)(\frac{1}{a_3}-1)(\frac{1}{a_4}-1)(\frac{1}{a_5}-1) \geq 1024.$$

Problem 3. A polynomial of degree 11 whose 11 roots form an arithmetic progression was written on a piece of paper. Due to an unfortunate accident most of the paper was lost and only the first three terms of the polynomial survieved: $x^{11} + 6x^{10} + 5x^9 + ...$ Find all the roots of this polynomial.

Problem 4. Find all polynomials whose all coefficients belong to the set $\{-1, 1\}$ and whose all roots are real.

Problem 5. Let $P(x)$ be a polynomial of degree n with n pairwise distinct roots $x_1, ..., x_n$. Prove that

$$\frac{P''(x_1)}{P'(x_1)}+\frac{P''(x_2)}{P'(x_2)}+...+\frac{P''(x_n)}{P'(x_n)}=0.$$

Problem 6. The polynomial $p(x) = x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n$ has n positive roots. Prove that

$$\frac{|a_1|}{\binom{n}{1}} \geq \frac{\sqrt{|a_2|}}{\binom{n}{2}} \geq \frac{\sqrt[3]{|a_3|}}{\binom{n}{n}} \geq ... \geq \frac{\sqrt[n]{|a_n|}}{\binom{n}{n}}.$$

Happy Thanksgiving!