

## Homework

due on Friday, September 25

**Problem 1.** Two rivers run parallel 2 miles apart. Two cities  $A$  and  $B$  lie between the rivers; each city is equidistant from the rivers and the cities are 3 miles apart. A scientist wishes to travel from  $A$  to  $B$ , collecting a sample of water from each river during his journey. What is the length of the shortest path he can follow. Justify your answer.

**Problem 2.** a) Show that if  $d$  is an integer not divisible by 3 then  $d^2 - 1$  is divisible by 3.

b) Show that if  $d$  is odd then  $d^2 - 1$  is divisible by 8.

c) Show that if  $4|n$  then  $n - 1$  is not a square.

d) Let  $n$  be a positive integer divisible by 24. Let  $s$  be the sum of all positive divisors of  $n - 1$ . Prove that  $s$  is divisible by 24.

**Problem 3.** Let  $f(x) = \frac{1}{1-x}$ . Denote by  $f^r$  the composition of  $f$  with itself  $r$  times. For example,  $f^2(x) = f(f(x))$  and  $f^5(x) = f(f(f(f(f(x)))))$ . Compute  $f^{2009}(2010)$ .

**Problem 4.** Let  $x > -1$  be a real number different from 0. Prove using induction that if  $n \geq 2$  is an integer then

$$(1+x)^n > 1+nx.$$