Homework

due on Wednesday, October 28

Problem 1. Initially, we are given the sequence $1, 2, ...100$. Every minute, we erase some two numbers u and v and replace them with the number $uv + u + v$. Clearly, we will be left with just one number after 99 minutes. What is this number? Justify your answer. Hint: Factor $1 + x + y + xy$.

Problem 2. Start with the set $\{3, 4, 12\}$. You are then allowed to replace any two numbers a and b with the new pair $0.6a - 0.8b$ and $0.8a + 0.6b$. Is it possible to perform a sequnce of such replacements to obtain the set $\{4, 6, 12\}$? Hint: What is $0.8^2 + 0.6^2$?

Problem 3. Prove that there are no positive integers a, b, c, d such that $a^2 + b^2 = 3(c^2 + d^2)$. Hint: What can you say about divisibility of a and b by 3?

Problem 4. Every participant of a tournament plays with every other participant exactly once. No game is a draw. After the tournament , every player makes a list with the names of all the players, who either were beaten by him or were beaten by a player beaten by him. Prove that there is a player whose list contains the names of all other players.

Problem 5. There are 2n people attending a meeting. Each person knows at least n other participants. Show that it is possible to accommodate the participants in n rooms so that each room is occupied by two participants who know each other.