Homework due on Monday, November 9

Problem 1. Find N for which the following is true: if there are 4 topics such that any two of a group of N people share interest in one of the topics then there are three people who all are interested in the same topic. Justify your answer.

Problem 2. Find smallest number f(n) for which the following statement is true: any graph with 2n + 1 vertices and f(n) edges contains three vertices such that any two of them are joined by an edge (i.e. it contains circuit of length 3). Remark: In class we showed that for graphs with even number 2n of vertices the analogous number is $n^2 + 1$.

Problem 3. There is exactly one road joining any 2 of n cities. Due to various constructions, each of these roads is a one way road. Suppose that whenever we can travel the road from city A to city B and the road from city B to city C then we can also travel the road from C to A. What can you say about n?

Problem 4. There are 10 people in a party. prove that either there are three people who are mutual starngers or there are 4 people who know each other. Is the same true for 9 people?