Homework

due on Monday, November 30

Problem 1. Prove that $1^{2009} + 2^{2009} + ... + n^{2009}$ is divisible by n iff n is odd or $4|n$. Hint: Try something like "Gauss' trick".

Problem 2. Let $a > 1$ be an even integer. Prove that the numbers $a^{2^n} + 1$, $n = 1, 2, ...$ are pairwise relatively prime. Conclude that there are infinitely many primes number. What can you say when a is odd?

Problem 3. Let p be a prime number and a an integer not divisible by p.

a) Show that there exists k such that $p|(a^k + 1)$ iff $\mathrm{ord}_p(a)$ is even.

b) Let $\mathrm{ord}_p(a)$ be even. Let $k > 0$ be a smallest number such that p divides $a^k + 1$. Prove that $\mathrm{ord}_p(a) = 2k$. Prove furthermore that p divides $a^n + 1$ iff $k|n$ and $2k$ does not divide n.

Problem 4. 1) Prove that if a prime p divides $2^n - 1$ then n has a prime divisor smaller than p.

2) Suppose that the natural numbers n_1, n_2,...,n_k satisfy

$$n_1|2^{n_2} - 1,\ n_2|2^{n_3} - 1,\ ...,\ n_{k-1}|2^{n_k} - 1,\ n_k|2^{n_1} - 1.$$

Prove that $n_1 = n_2 = ... = n_k = 1$.

Problem 5. Find the last digit of $\left\lfloor \dfrac{10^{20000}}{10^{100} + 3} \right\rfloor$.