Homework

due Wednesday, October 20

Problem 1. Let f be a polynomial such that $f(x) - 2f'(x) + f''(x) \geq 0$ for all x. Prove that $f(x) \geq 0$ for all x.

Problem 2. Prove by induction that 5 divides $n^5 - n$ for every natural number n.

Problem 3. Bay Area Rapid Food sells chicken nuggets. You can buy packages of seven or packages of 11. What is the largest integer n such that there is no way to buy exactly n nuggets? Can you generalize this?

Problem 4. Compute $\int_0^2 \frac{x^3 dx}{6x^2 - 12x + 8}$. Hint: expand $(x-2)^3$?

Problem 5. Prove that $1^{2011} + 2^{2011} + \ldots + n^{2011}$ is divisible by n if and only if n is odd or $4|n$.

Problem 6. There are 2010 people sittng at a round table (a really big one). Each peson is given a number which is equal to the avarage of the numbers given to his/her two neighbours. Prove that each person is given the same number.