## Homework

due on Wednesday, November 3 $\,$ 

**Problem 1.** Prove that among any five points selected inside an equilateral triangle with side equal to 2, there always exists a pair at the distance not greater than 1.

**Problem 2.** 21 integers are selected from  $\{1, 2, 3, ..., 400\}$ . Prove that two of them, say x and y, satisfy  $0 < |\sqrt{x} - \sqrt{y}| < 1$ .

**Problem 3.** Nine distinct points with all coordinates integral are selected in the space. Prove that the line segment with ends at certain two of these points contains in its interior a point with all coordinates integral.

**Problem 4.** 2004 flies are inside a cube of side 1. Show that some 3 of them are within a sphere of radius 1/11.

**Problem 5.** Six points are positioned in a  $3 \times 4$  rectangle. Prove that there are two of them at the distance not exceeding  $\sqrt{5}$  from each other.

**Problem 6.** Let  $a_1, a_2, \ldots, a_n$  be positive integers all of whose prime divisors are  $\leq 13$ .

a) Show that if  $n \ge 65$  then there exist two of these integers whose product is a perfect square.

b) Show that if  $n \ge 193$  then there exists four of these integers whose product is a perfect fourth power.

Hint Use a) to get many pairs of numbers which multiply to a square. Use a) again to get two disjoint such pairs with square roots of products in each pair multiplying to a square.