Exam 1, Math 488A/Math 575P
November 25, 2010

In order to get full credit a correct solution must be written carefully, with detailed explanation of all steps. Each problem is worth 10 points.

Problem 1. Prove that there is no integer $n > 2$ such that $n(n+6)$ is a square of an integer.

Problem 2. A set of 10 different numbers is selected from $\{1, 2, \ldots, 18\}$. Prove that among the selected integers there are two numbers which differ by 3.

Problem 3. Let n be a positive integer. Let $d_1, \ldots, d_k$ be all divisors of n. Prove that the number

$$\frac{2}{\ln n} \sum_{i=1}^{k} \ln d_i = \frac{2}{\ln n}(\ln d_1 + \ln d_2 + \ldots + \ln d_k)$$

is an integer.

Problem 4. Compute the integrals

$$\int_0^\pi \frac{x^2 \sin x}{x^2 + (\pi - x)^2} dx,$$

$$\int_0^\pi \frac{x^3 \sin x}{3x^2 - 3\pi x + \pi^2} dx.$$