**Homework**

due on Wednesday, November 30

**Problem 1.** There is exactly one road joining any two of $n$ cities. Due to various constructions, each of these roads is a one way road. Suppose that whenever we can travel the road from city $A$ to city $B$ and the road from city $B$ to city $C$ then we can also travel the road from $C$ to $A$. What can you say about $n$?

**Problem 2.** Find smallest number $f(n)$ for which the following statement is true: any graph with $2n+1$ vertices and $f(n)$ edges contains three vertices such that any two of them are joined by an edge (i.e. it contains circuit of length 3). Remark: In class we showed that for graphs with even number $2n$ of vertices the analogous number is $n^2+1$.

**Problem 3.** a) There are 10 people in a party. Prove that there are either 3 people who are mutual strangers or there are 4 people who know each other.

b)) Prove that the same is true when 10 is replaced by 9. Hint: Show that if there is a person who knows at most 4 other people then the result holds. Show that if there is a person who knows more than 5 other people then the result holds (you can use what we did in class about 6 people discussing 2 topics). Prove that there must be a person for which one of the above is true.

**Problem 4.** Each of the three companies employs $n$ people. Each employee knows exactly $n+1$ employees from other two companies. Prove that one can choose one employee from each company so that the three know each other.

**Problem 5.** Any group of four people in a village contains a person who has a common interest with each of the other three poeple. Assuming that the village has at least 4 poeple prove that there is a person sharing a common interest with everybody else. .

**Problem 6.** a) A graph is called **bipartite** if its vertices can be divided into two disjoint groups such that no two vertices in the same group are connected by an edge. Prove that a graph is bipartate iff it does not have a circuit (closed path) with odd number of vertices.

b) $N$ cities are served by $n$ airlines. For any two cities there is at lest one airline

offering a direct connection between these cities (all airlines provided service in both directions). Suppose that $N > 2^n$. Prove that there is an airline offering a roun trip with an odd number of landings. Hint: Use induction on $n$ and a).