## Exam 1, Math 488A/Math 575P November 2, 2010

In order to get full credit a correct solution must be written carefully, with detailed explanation of all steps. Each problem is worth 10 points.

**Problem 1.** Prove that there is no integer n > 2 such that n(n+6) is a square of an integer.

**Problem 2.** Compute the integral

$$\int_0^1 \frac{(2x^2+1)\sin \pi x}{x^2-x+1} dx.$$

Be very careful with your algebra. Note:  $\sin(\pi - t) = \sin t$ .

## Solve one of the following 2 problems.

**Problem 3.** Let *n* be a positive integer. Let  $d_1, \ldots, d_k$  be all divisors of *n*. Prove that the number

$$\frac{2}{\ln n} \sum_{i=1}^{k} \ln d_i = \frac{2}{\ln n} (\ln d_1 + \ln d_2 + \ldots + \ln d_k)$$

is an integer.

**Problem 4.** Compute the sum

$$\sum_{k=0}^{2010} \frac{\sin^{2011} \frac{k\pi}{4020}}{\sin^{2011} \frac{k\pi}{4020} + \cos^{2011} \frac{k\pi}{4020}}$$

Solve one of the following 2 problems.

**Problem 5.** There are 28 points selected in an equilateral triangle with side of length 3. Prove that there are 4 among these points which are within distance 1 to each other.

**Problem 6.** A set of 10 different numbers is selected from  $\{1, 2, ..., 18\}$ . Prove that among the selected integers there are two numbers which differ by 3.