Homework
due on Thursday, February 18

Read carefully the first three chapters of Dunham's book and chapters 12 and 14 int he book by Berlinghoff and Gouvea. Solve Question 4 to Chapter 12 and Question 4 to Chapter 14. Also solve the following problems.

Problem 1. We have seen that in Euclid's *Elements* the following result is proved:

Theorem 1. *Let n be a positive integer such that $2^{n+1} - 1$ is a prime number. Then the number $2^n(2^{n+1} - 1)$ is perfect.*

a) Prove this theorem (Hint: list all proper divisors of $2^n(2^{n+1} - 1)$).

b) Prove conversely, that if a number of the form $2^n(2^{n+1} - 1)$ is perfect then $2^{n+1} - 1$ is a prime number.

c) Prove that if $2^m - 1$ is a prime then so is m (Hint: do a proof by contrapositive, i.e. show that if m is composite then so is $2^m - 1$).

Problem 2. Here is quadrature of the second lune considered by Hippocrates.

a) Construct a trapezium $ABCD$ such that $AB = \sqrt{3}$, $BC = CD = DA = 1$. The construction should be given as a "recipe" followed by explanation and justification of each step. (Use 1 inch or 3 cm as a unit).

b) Construct the circle c_1 circumscribed about $ABCD$. Let O be its center.

c) Construct a point E on the opposite side of the line AB than C such that the triangles EAB and OCD are similar. Let c_2 be the circle with center E and radius EA. Prove that the lune determined by circles c_1 and c_2 has area equal to the area of $AEBO$.

d) Perform quadrature of $AEBO$.

Problem 3. Consider an isosceles triangle with base of length 18 and height of length 16. Divide this triangle into several polygonal pieces from which a square of side 12 can be assembled (use 1 cm as a unit). Explain your solution carefully.