Homework

due on Thursday, April 22

Read carefully chapter 5 of Dunham's book and Chapters 13 and 18 in the book by Berlinghoff and Gouvea. Do project 1 of Chapter 13, question 2 and project 3 of Chapter 18. Also solve the following problems.

Problem 1. Learn a proof of the Theorem of Menalaus (for example, from one of the links provided on the course web page). Use this theorem to prove the following results

a) The lines AA_1, BB_1 and CC_1 intersect in one point O. Let the lines AB and A_1B_1 intersect at C_2, the lines AC and A_1C_1 intersect at B_2, and the lines BC and B_1C_1 intersect at A_2. Prove that the points A_2, B_2, C_2 are collinear. This is often called Desarques Theorem. Hint. Apply Menelaus' Theorem to triangles OAB, OBC, OAC and appropriate lines. Then apply its converse to the traiangle ABC.

b) Points A_1, B_1, C_1 are collinear and so are points A_2, B_2, C_2. The lines A_1B_2 and A_2B_1 intesect at a point C, the lines A_1C_2 and A_2C_1 intesect at a point B, and the lines B_1C_2 and B_2C_1 intesect at a point A. Prove that the points A, B, C are collinear. This is Pappus' Theorem. Hint. Let A_0, B_0, C_0 be the vertices of the triangle determined by the lines A_1B_2, B_1C_2, and C_1A_2 (where A_0 is the point of intersection of A_1B_2 and A_2C_1, etc.). Apply Menalaus' Theorem to the triangle $A_0B_0C_0$ and five appropriate lines.

Problem 2. Justify the Newton's solution of the duplication of a cube, as described in the link on the course web page, by following the following hints (the notation is from the link). Alternatively, find your own solution.

a) Let $AG = x$, $CG = y$. Show that $\angle ACG = \pi/2$ and derive a relation between x and y.

b) Show that $DC = \sqrt{3}$.

c) Apply the Theorem of Menelaus to the triangle ADG. This should give you another relation between x and y.

d) Find x and y.