Homework
due on Thursday, February 16

Read carefully Chapters 3 and 4 of the book by Joseph. Solve the following problems.

Problem 1. Write $7217\frac{2}{15}$ using Babylonian sexagesimal notation and Egyptian hieroglyphic notation.

Problem 2. One possible method the Babylonians might have used to approximate the square root of b is the following: start with an approximation a_1 to $\sqrt{b}$. Then take for a new approximation $a_2 = (a_1 + b/a_1)/2$. And keep doing it, i.e you define recursively a sequence of approximations by the formula

$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{b}{a_n}\right).$$

a) Starting with $b = 5$ and $a_1 = 2$ compute a_3 in sexagesimal system and in decimal system. How many decimal digits agree with the actual value of $\sqrt{5}$?

b) In general, assuming that the sequence a_n converges, prove that its limit is $\sqrt{b}$.

c) (Extra credit). Prove that $a_n \geq \sqrt{b}$ for all $n \geq 2$. Then prove that $a_{n+1} \leq a_n$ for all $n \geq 2$ and conclude that a_n converges. This proves that the sequence a_n consists indeed of better and better aproximations to $\sqrt{b}$.

Problem 3. a) Use the greedy algorithm to find Egyptian fraction for 24/29.

b) (Extra credit) Find an Egyptian fraction for 24/29 with all the denomiators even and less than 250. Explain how you did it.

Problem 4. Assume the following result: the volume of a pyramid (or cone) with base of area A (any shape) and height H is equal to $AH/3$. Suppose now that such a pyramid is truncated by a plane parallel to the base whose distance to the base is h and let B be the area of the top of the truncated pyramid. Prove that the volume V of the truncated pyramid is given by the following formula:

$$V = \frac{1}{3}h(A + \sqrt{AB} + B).$$

Explain carefully your argument using full sentences. Hint. Use the facts that the two pyramids determined by the top and bottom planes are similar.