

## Homework

due on Thursday, March 1

Read carefully the first three chapters of Dunham's book. Solve the following problems.

**Problem 1.** As discussed in class, Pythagoras and his follower are credited with the study of perfect numbers. In Euclid's *Elements* the following result is proved:

*Theorem 1. Let  $n$  be a positive integer such that  $2^{n+1} - 1$  is a prime number. Then the number  $2^n(2^{n+1} - 1)$  is perfect.*

- a) Prove this theorem (Hint: list all proper divisors of  $2^n(2^{n+1} - 1)$ ).
- b) Prove conversely, that if a number of the form  $2^n(2^{n+1} - 1)$  is perfect then  $2^{n+1} - 1$  is a prime number.
- c) Prove that if  $2^m - 1$  is a prime then so is  $m$  (Hint: do a proof by contrapositive, i.e. show that if  $m$  is composite then so is  $2^m - 1$ ).

**Problem 2.** Here is quadrature of the second lune considered by Hippocrates.

- a) Construct a trapezium  $ABCD$  such that  $AB = \sqrt{3}$ ,  $BC = CD = DA = 1$ . The construction should be given as a "recipe" followed by explanation and justification of each step. (Use 1 inch or 3 cm as a unit).
- b) Construct the circle  $c_1$  circumscribed about  $ABCD$ . Let  $O$  be its center.
- c) Construct a point  $E$  on the opposite side of the line  $AB$  than  $C$  such that the triangles  $EAB$  and  $OCD$  are similar. Let  $c_2$  be the circle with center  $E$  and radius  $EA$ . Prove that the lune determined by circles  $c_1$  and  $c_2$  has area equal to the area of  $AEBO$ .
- d) Perform quadrature of  $AEBO$ .

**Problem 3.** Consider an isosceles triangle with base of length 18 and height of length 16. Divide this triangle into several polygonal pieces from which a square of side 12 can be assembled (use 1 cm as a unit). Explain your solution carefully.