# Homework

due on Thursday, March 6

Rad carefully the first three chapters of Dunham's book. Solve the following problems.

**Problem 1.** As discussed in class, Pythagoras and his follower are credited with the study of perfect numbers. In Euclid's *Elements* the following result is proved:

*Theorem* 1. *Let $n$ be a positive integer such that $2^{n+1}-1$ is a prime number. Then the number $2^n(2^{n+1}-1)$ is perfect.*

a) Prove this theorem (Hint: what are the proper divisors of $2^n p$ when $p$ is a prime number ? what is their sum ?).

b) Prove conversely, that if a number of the form $2^n(2^{n+1}-1)$ is perfect then $2^{n+1}-1$ is a prime number.

c) Prove that if $2^m-1$ is a prime then so is $m$ (Hint: do a proof by contrapositive, i.e. show that if $m$ is composite then so is $2^m-1$).

**Problem 2.** Here is quadrature of the second lune considered by Hippocrates.

a) Construct a trapezium $ABCD$ such that $AB=\sqrt{3}$, $BC=CD=DA=1$. The construction should be given as a "recipe" followed by explanation and justification of each step. (Use 1 inch or 3 cm as a unit).

b) Construct the circle $c_1$ circumscribed about $ABCD$. Let $O$ be its center.

c) Construct a point $E$ on the opposite side of the line $AB$ than $C$ such that the triangles $EAB$ and $OCD$ are similar. Let $c_2$ be the circle with center $E$ and radius $EA$. Prove that the lune determined by circles $c_1$ and $c_2$ has area equal to the area of $AEBO$.

d) Perform quadrature of $AEBO$.

**Problem 3.** Consider an isosceles triangle with base of length 18 and height of length 16. Divide this triangle into several polygonal pieces from which a square of side 12 can be assembled (use 1 cm as a unit). Explain your solution carefully and provide the actual pieces made out of a thin cardboard (or paper).