Homework

due on Monday, February 29

Rad carefully the first three chapters of Dunham's book. Solve the following problems.

Problem 1. Here is quadrature of the second lune considered by Hippocrates.

a) Construct a trapezium $ABCD$ such that $AB = \sqrt{3}$, $BC = CD = DA = 1$. The construction should be given as a "recipe" followed by explanation and justification of each step. (Use 1 inch or 3 cm as a unit).

b) Construct the circle c_1 circumscribed about $ABCD$. Let O be its center.

c) Construct a point E on the opposite side of the line AB than C such that the triangles EAB and OCD are similar. Let c_2 be the circle with center E and radius EA. Prove that the lune determined by circles c_1 and c_2 has area equal to the area of $AEBO$.

d) Perform quadrature of $AEBO$.

Problem 2. Consider an isosceles triangle with base of length 18 and height of length 16. Divide this triangle into several polygonal pieces from which a square of side 12 can be assembled (use 1 cm as a unit). Explain your solution carefully and provide the actual pieces made out of a thin cardboard (or paper).

Problem 3. In this problem we work out an equation of a quadratrix in a convenient coordiante system.

Recall the construction of the quadratrix: a line p is originally alligned with the x-axis and revolves counterclockwise about the point $(0, 0)$ with constant angular velocity. At the same time the line q is aligned with the line $x = 1$ and it slides along the x-axis to the left with constant velocity. The velocities are chosen so that both lines coincide with the y-axis at the same time (when p revolves by $\pi/4$). The point of intesection of the two moving lines traces a curve called **quadratrix**.

a) Let $A = (x, y)$ be a point on the quadratrix $(0 < x < 1)$ and let α be the angle beteen the x-axis and the line OA, where $O = (0, 0)$. Prove that $x = 1 - \dfrac{2\alpha}{\pi}$.

b) With the notation as in a), prove that $y = x\cot(\pi x/2)$. This is the equation of our quadratrix in the chosen coordinate system. Sketch the graph of the quadratrix.

c) Use b) and some basic calculus to show that the quadratrix intersects the y-axis at the point $(0, 2/\pi)$ (why can't you just plug-in $x = 0$?).