

Homework 4
due Monday, November 4

Problem 1. Prove that among any five points selected inside an equilateral triangle with side equal to 2, there always exists a pair at the distance not greater than 1.

Problem 2. 21 integers are selected from $\{1, 2, 3, \dots, 400\}$. Prove that two of them, say x and y , satisfy $0 < |\sqrt{x} - \sqrt{y}| < 1$.

Problem 3. Nine distinct points with all coordinates integral are selected in the space. Prove that the line segment with ends at certain two of these points contains in its interior a point with all coordinates integral.

Problem 4. 2004 flies are inside a cube of side 1. Show that some 3 of them are within a sphere of radius $1/11$.

Problem 5. Let a_1, a_2, \dots, a_n be positive integers all of whose prime divisors are ≤ 13 .

a) Show that if $n \geq 65$ then there exist two of these integers whose product is a perfect square.

b) (extra credit) Show that if $n \geq 193$ then there exists four of these integers whose product is a perfect fourth power.

Hint Use a) to get many pairs of numbers which multiply to a square. Use a) again to get two disjoint such pairs a, b and c, d such that $\sqrt{ab}\sqrt{cd}$ is a square.