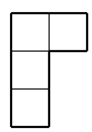
Homework

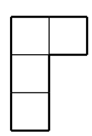
due on Wednesday, November 20

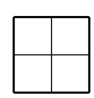
Problem 1. A 23×23 square is tiled by 1×1, 2×2, and 3×3 tiles. Prove that at least one 1×1 tile must be used . Find such a tiling with exactly one 1×1 tile. Hint: put a number in each 1×1 square of the big sqaure so that 2×2 and 3×3 tiles cover a total divisible by 3.

Problem 2. Consider an $n \times n$ chessboard with all 4 corner squares removed. Prove that if the board can be covered with $L-$tetrominoes then $n - 2$ is a multiple of 4. Is the converse true? (an $L-$tetromino is a plane figure shown below, constructed from four unit squares arranged in the form of L)



Problem 3. Prove that an 8×8 board cannot be covered by 15 $L-$tetrominos and one square tetromino (an $L-$tetromino is a plane figure shown below, constructed from four unit squares arranged in the form of L; a square tetromino is a plane figure shown below, constructed from four unit squares arranged in the form of a square)





Problem 4. Numbers $1, 2, 3, ..., 2014$ are written on a blackboard. Every now and then somebody picks two numbers a and b and replaces them by $a - 1$, $b + 3$. Is it possible that at some point all numbers on the blackboard are even? Can they all be odd?

Problem 5. Prove that there are no positive integers a, b, c, d such that $a^2 + b^2 = 3(c^2 + d^2)$. Hint: What can you say about divisibility of a and b by 3? Look at solution with smallest possible a.

Problem 6. Every participant of a tournament plays with every other participant exactly once. No game is a draw. After the tournament , every player makes a list with the names of all the players, who either were beaten by him or were beaten by a player beaten by him. Prove that there is a player whose list contains the names of all other players.