Homework

due on Wednesday, December 11

Read cerefully section 5.5 in the book. 5.5.39, 5.5.43, 5.5.47, 5.5.48 (hint: if $a < b$ and $x > y$ then $ax + by < ay + bx$) . Also solve the following problems.

Problem 1. Solve problem 5.5.34 as follows: first do case $n = 2$ algebracially (there is a geometric interpretation; do you see it?); then do induction on n.

Problem 2. Let a, b, c, d be positive numbers. Prove that

$$\frac{a+b+c+d}{4} \geq \sqrt{\frac{ab+ac+ad+bc+bd+cd}{6}}.$$

Hint: Write the sum on the right side in terms of $a^2+b^2+c^2+d^2$ and $a+b+c+d$. Another way: use $(x-y)^2 \geq 0$.

Problem 3. Positive numbers a, b, c satisfy $a^{-1} + b^{-1} + c^{-1} = 3$.

a) Prove that $abc \geq 1$

b) Prove that $(a+b)(a+c)(b+c) \geq 8$. When does the equality hold?

Problem 4. Each of the three companies employs n people. Each employee knows exactly $n+1$ employees from other two companies. Prove that one can choose one employee from each company so that the three know each other.

Problem 5. Any group of four people in a village contains a person who has a common interest with each of the other three poeple. Assuming that the village has at least 4 poeple prove that there is a person sharing a common interest with everybody else. .