

Homework 1

due on Monday, September 15

Solve the following problems. Write the solutions carefully, using full sentences which clearly explain your reasoning (writing an exposition of your solution is as important as finding the solution).

Problem 1. A certain young person was asked about her birthday (this happened around 1950). *I was born on September 6, she said, but I celebrate my birthday not every year, but every 1000 days. I find this much more convenient. My last celebration was on August 1, 1948.*

What year was she born?

Problem 2. Three scholars are seated in a row one behind the other. They are shown three red hats and two blue hats. Each is blindfolded, a hat is placed on each head, and the blindfolds are removed. Starting from the back, each scholar is then asked what color hat was on his or her head, and to explain how they knew for certain the hat was that color.

The scholar in the back, who could see two hats, could not answer. The scholar in the middle, who could see one hat, could not answer either. The scholar in the front, who could see no one's hat, was able to answer correctly, and explain how she knew the color for certain.

What color hat did the scholar in front have on her head, and how was she able to answer correctly?

Problem 3. There are 100 coins divided into 10 stacks, with 10 coins in each stack. There are 90 genuine coins and 10 fake coins. A genuine coin weighs 2 ounces and a fake coin weighs 1 ounce. The 10 fake coins are contained in one of the stacks.

You have a scale with which you can weigh any number of coins. Using only one weighing, describe how you would determine which stack contains the fake coins.

Problem 4. 16 seemingly identical coins are given. It is suspected that some (but not all) of the coins are fake. It is known that all the fake coins (if any) are identical and have weight different from the genuine coins. Find a procedure to determine if there are any fake coins using smallest possible number of weighings on a balanced scale (you do not need to find the fake coins). What about the same problem with

17 coins?