## Homework 4 due on Wednesday, November 5

**Problem 1.** 2004 flies are inside a cube of side 1. Show that some 3 of them are within a sphere of radius 1/11.

**Problem 2.** Six points are positioned in a  $3 \times 4$  rectangle. Prove that there are two of them at the distance not exceeding  $\sqrt{5}$  from each other.

**Problem 3.** Let  $a_1, a_2, \ldots, a_n$  be positive integers all of whose prime divisors are  $\leq 13$ .

a) Show that if  $n \ge 65$  then there exist two of these integers whose product is a perfect square.

b) (extra credit) Show that if  $n \ge 193$  then there exists four of these integers whose product is a perfect fourth power.

**Hint** Use a) to get many pairs of numbers which multiply to a square. Use a) again to get two disjoint such pairs a, b and c, d such that  $\sqrt{ab}\sqrt{cd}$  is a square.

**Problem 4.** Prove that for every natural number n we have

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$$2\sqrt{n} - 1 \ge \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} \ge 2\sqrt{n+1} - 2.$$
  
Hint.  $\sqrt{x+1} - \sqrt{x} = \frac{1}{\sqrt{x} + \sqrt{x+1}}.$