Homework 2

due on Wednesday, February 24

Read carefully the first 3 chapters in the book.

Problem 1. a) Find all subgroups and normal subgroups of S_3 and S_4. Identify the derived group and the center.

b) In S_n let $S(i)$ be the subgroup which consists of all permutations which fix i and set $A(i) = A_n \cap S(i)$. Prove that $S(i)$, $A(i)$ are maximal subgroups of S_n, A_n respectively.

Remark. A subgroup H of a group G is called **maximal** if it is not contained in any larger, proper subgroup of G, i.e. if H is a maximal element in the set of all proper subgroups of G ordered by inclusion.

c) solve Problems 4, 12 to section 3.5.

Problem 2. Let G be a group with center $Z(G)$. Prove that if $G/Z(G)$ is cyclic then G is abelian

Problem 3. a) Prove the Second Homomorphism Theorem as stated in the notes.

b) We say that two subgroups K, H of a group G are **conjugate** iff there is $g \in G$ such that $gKg^{-1} = H$. Prove that in the Correspondence Theorem two subgroups of G which contain $\ker f$ are conjugate iff their images in H are conjugate.

Solve problems 3,8,10 to 3.3, problem 21 to 3.2, problem 42 to 3.1, problem 14 c, d to 2.4, problem 26 to 2.3, problem 9 to 1.6, problems 12, 15 to 1.3 and problems 25, 31 to 1.1.