Homework 3

due on Monday, March 7

Read carefully chapter 4 in the book.

Problem 1. Let G be a finite group and p a prime number such that $p||G|$. Consider the set S of all $p-$tuples $(a_1, ..., a_p)$ of elements from G such that $a_1 a_2 ... a_p = e$, i.e.

$$S = \{(a_1, ..., a_p) : a_i \in G \text{ for all } i, \text{ and } a_1 ... a_p = e\}$$

Let C be a cyclic group of order p and f a generator for C. We define an action of C on S as follows: if $s \in C$ then $s = f^i$ for a unique $0 \leq i < p$ and we set $s * (a_1, ..., a_p) = (a_{i+1}, a_{i+1}, ... a_p, a_1, a_2, ..., a_i)$.

a) Check that this is indeed an action of C on S.

b) Show that the number of elements in S equals $|G|^{p-1}$.

c) Show that each fixed point for this action is of the form $(g, ..., g)$ for some $g \in G$ such that $g^p = e$.

d) Conclude that G has a nontrivial element of order p (so we get a different proof of Cuchy's theorem).

Solve problems 7,9 to 4.1, problems 7, 12 to 4.2, problems 23,24,26,27 to 4.3 (problem 24 follows easily from 23 and 26, 27 follow from 24), 12,18 to 4.4