Homework 2

Problem 1. Let M be a trivial G-module (i.e. an abelian group with trivial action of G). Show that $H_1(G, M)$ is isomorphic to $G^{ab} \otimes_{\mathbb{Z}} M$, where $G^{ab} = G/[G, G]$ is the abelianization of G.

Problem 2. Let M be a trivial RG-module.

a) Show that $I \otimes_{RG} M$ is isomorphic to $I/I^2 \otimes_R M$.

b) Show that $Hom_{RG}(I, M)$ is isomorphic to $Hom_R(I/I^2, M)$.

c) Show that $Hom_R(I/I^2, R)$ is isomorphic to $Ext^1_{RG}(R, R)$.

   Read carefully Chapter III of Brown's book (you should also be familiar with the content of Chapters I and II). Solve Probelm 1 to III.8, Problems 2 and 5 to III.5, Problem to III.4, Problems 3 and 4 to III.3, Problems 1 and 3 to III.1.