Math 603A, Fall 2009

Algebraic Number Theory

MWF 10:50-11:50, T 10:05-11:30

Marcin Mazur
Office: LN 2228

E-mail: mazur@math.binghamton.edu

The goal of this course is to introduce basic concepts and results of Algebraic Number Theory. This theory belongs to the greatest achievements of mathematical thought and , in my opinion, should be a part of a mathematical education of every mathematician.

Among the topics discussed in class will be the following:

- the rings of algebraic integers; units and class groups;
- valuation theory; local fields;
- extensions of local and global fields; different and discriminant;
- ramification theory;
- quadratic and cyclotomic fields;
- zeta functions and L-series

The main texts for the course are:

- D. Marcus, Number Fields, Universitext, Springer-Verlag.
- Helmut Koch, Number Theory: Algebraic Numbers and Functions, Graduate Studies in Mathematics, Vol. 24, AMS 2000.

Note however that the order in which the material is introduced and the presentation of many topics in class will differ from the approach taken in the above books and further literature will be provided (it is important to get familiar with many different approaches to the subject). There are many excellent books about algebraic number theory. Among them I particularly like the following:

• W. Narkiewicz, Elementary and Analytic Theory of Algebraic Numbers. This is a great, encyclopedic account, a great reference to own.

- Z. I. Borevich, I. R. Shafarevich, **Number Theory**. A beautiful introduction to the subject, form a slightly different angle.
- K. Ireland, M. Rosen, A Classical Introduction to Modern Number Theory, Graduate Texts in Mathematics, v. 84. Very accessible, contains many topics not treated in other introductory books..
- H. Weyl, **Algebraic Theory of Numbers**. A bit outdated approach, well worth reading. Provides motivation for many basic concepts. It is a good place to see how the subject looked 70 years ago.
- J. Neukirch, **Algebraic Number Theory**, Grundlehren Der Mathematischen Wissenschaften, Vol. 322, Springer-Verlag 1999. A great, modern approach to algebraic number theory.
- H. Hasse, **Number Theory**. For many years it was the main book for the subject.
- A. Weil, **Basic number theory**. Despite the title, it is a very demanding book, introducing the subject from completely different (but very important) points of view.

I will assume a decent familiarity with linear algebra (Math 507) and with basic concepts of algebra. More specialized topics from Math 525, Math 503, Math 504 will be very helpful but not necessary to follow the class.