

Homework

due on Monday, July 19

Read carefully Chapters 6 and 9. Solve problems 9, 18 to Chapter 6, problems 18, 20 to chapter 9. Also solve the following problems.

Problem 1. Let p, q be distinct prime numbers. Prove that

$$p^{q-1} + q^{p-1} \equiv 1 \pmod{pq} .$$

Problem 2. Let m, n be positive integers such that $m|n$. Prove that $\phi(m)|\phi(n)$ and that $\phi(mn) = m\phi(n)$

Problem 3. Compute $\phi(2592)$, $\phi(111111)$, $\phi(15!)$.

Problem 4. Use Euler Theorem to find the remainder upon division of n by m , where

a) $n = 29^{202}$, $m = 13$;

b) $n = 99^{999999}$, $m = 23$

c) $n = 29^{198}$, $m = 20$

d) $n = 3^{1000000}$, $m = 14$

Problem 5. Prove that if n is relatively prime to 72 then $n^{12} \equiv 1 \pmod{72}$.