

Homework 1
due on Wednesday, July 13

Read carefully sections 1.1-1.5 in Stoll's book and section 1 of Chapter 1 in Shen's book. Read carefully the notes on sets linked on the course web page. Solve the following problems.

Problem 1. Let A, B, C be sets. Prove the following equalities:

- a) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.
- b) $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap B \cap C)$.
- c) $(A \setminus B) \setminus C = A \setminus (B \cup C)$.
- d) $(A \Delta B) \Delta (A \cap B) = A \cup B$.

Problem 2. Let A, B, C be sets.

- a) Build a membership table to prove the first De Morgan's law: $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.
- b) Prove the second De Morgan's law as follows. Express each side of the equality

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

in terms of the operations $+$, \cdot , where $+$ denotes symmetric difference and \cdot denotes intersection. Then use basic algebraic manipulations to prove that both sides are equal.

- c) Prove that $[A \setminus (B \Delta C)] \cap B = A \cap B \cap C$ in two ways using the method of a) and the method of b).