Homework 12

due on Friday, August 5

Read carefully sections 4,5 of Chapter 2 in Shen's book and and sections 2.6, 2.7 in Stoll's book. Solve the following problems.

Problem 1. Let $(A, \leq_A)$ be a well ordered set and let $B, \leq_B$ be a prtially ordered set. We deline a relation $\leq$ of B^A as follows. Let $f, g : A \longrightarrow B$ be elements of B^A. We say that $f \leq g$ if either $f = g$ or for the smallest $a \in A$ such that $f(a) \neq g(a)$ we have $f(a) \leq_B g(a)$ (note that such smallest a exists since A is well ordered).

a) Prove that $\leq$ is a parial order on B^A.

b) Suppose that $\leq_B$ is a linear order on B. prove that $\leq$ is a linear order of B^A.

c) Let $A = \mathbb{N}$ and $B = \{0, 1\}$. Prove that the order $\leq$ on B^A is not a well order. Hint: find an infinite decreasing sequence in B^A.

d) Suppose that $\leq$ is a well order. Prove that $\leq_B$ is a well order. Hint: prove that B is isomorphic to a subest of B^A. Prove that A is finite if B has more than one element. Hint: Use c).

Problem 2. Let R be a relation on a set A which is antisymmetric. Suppose that for any non-empty subset B of A there is $u \in B$ such that uRb for all $b \in B$. Prove that R is a well order.