## Homework 14 due on Thursday, August 11

Read carefully sections 8, 9 of Chapter 2 in Shen's book and and sections 2.8, 2.9, 2.10, 2.11 in Stoll's book. Solve the following problems.

**Problem 1.** Let  $(A, \leq_A)$  be a partially ordered set. We will think of partial orders as subsets of  $A \times A$  (as any relation is such a subset). Consider the family  $\mathcal{P}$  which cosists of partial orders R on A which contain the partial order  $\leq_A$ , i.e.  $R \in \mathcal{P}$  if and only if  $R \subseteq A \times A$  is a partial order and  $\leq_A \subseteq R$ . We define a partial order  $\leq$ on  $\mathcal{P}$  as follows:  $R_1 \leq R_2$  if and only if  $R_1 \subseteq R_2$ .

a) Prove that  $\mathcal{P}$  is not empty and it is inductively ordered by  $\leq$ .

b) Explain why  $(\mathcal{P}, \leq)$  has maximal elements. Let R be a maximal element. Prove that R is a linear order on A which contains  $\leq_A$ . Hint. Use Problem 2 from Homework 11.

Problem 2. Review, Review, Review, ... .