Homework 4

due on Monday, July 18

Read carefully sections 3,4,6,7 of Chapter 1 in Shen's book and and sections 2.3-2.4 in Stroll's book (2.1 and 2.2 are about natural numbers and induction; try to read them too, though he does much more that what we did in class). Solve the following problems.

Problem 1. Prove by induction on n that

$$A \setminus (A_1 \cup A_2 \cup \ldots \cup A_n) = (A \setminus A_1) \cap (A \setminus A_2) \cap \ldots \cap (A \setminus A_n).$$

Formualte and prove a similar generalization of the second De Morgan's law.

Problem 2. a) Let A, B, C, D be sets. Prove that $(A \cap C) \vartriangle (B \cap D) \subseteq (A \vartriangle B) \cup (C \vartriangle D)$.

b) Prove by induction on n that

$$(A_1 \cap A_2 \cap \ldots \cap A_n) \vartriangle (B_1 \cap B_2 \cap \ldots \cap B_n) \subseteq (A_1 \vartriangle B_1) \cup (A_2 \vartriangle B_2) \cup \ldots \cup (A_n \vartriangle B_n)$$

Problem 3. Let A be a set with an infinite countable subset B. Let C be a countable set such that $A \cap C = \emptyset$. Prove that A and $A \cup C$ have the same cardinality. Hint: Write $A = (A \setminus B) \cup B$. Construct a bijection from $(A \setminus B) \cup B \cup C$ onto $(A \setminus B) \cup B$.

Problem 4. Let T be a set of open intevals (i.e. subsets of the real numbers of the form (a, b) for some $a < b$) such that any two intervals in T are disjoint. Prove that T is countable. Hint: between any two real numbers there is a rational number.

Problem 5. This problem is **optional**, you may earn extra credit. Prove that for any natural number n, an elemant x belongs to the set $A_1 \vartriangle A_2 \vartriangle \ldots \vartriangle A_n$ if and only if x belongs to an odd number of the sets $A_1, A_2, \ldots, A_n$.