Homework 5

due on Wednesday, July 20

Read carefully sections 3,4,6,7 of Chapter 1 in Shen's book and and sections 2.3-2.4 in Stroll's book (2.1 and 2.2 are about natural numbers and induction; try to read them too, though he does much more that what we did in class). Solve the following problems.

Problem 1. Let $a < b$ and $c < d$ be real numbers. Construct a bijection between $[a, b)$ and $[c, d)$, and between $[a, b)$ and $(c, d]$. Prove that there is a bijection between (a, b) and $[a, b)$, and between (a, b) and $[a, b]$ (use the fact that intervals are infinite sets).

Problem 2. Prove that there is a bijection between $[0, 1] \cup [2, 3] \cup [4, 5] \cup \ldots$ and $[0, 1]$. Hint: $(0, 1] = (1/2, 1] \cup (1/3, 1/2] \cup (1/4, 1/3] \cup \ldots$.

Problem 3. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function. A number $x \in \mathbb{R}$ is called a strict local maximum of f if there is $\epsilon > 0$ such that for any h such that $0 < |x - h| < \epsilon$ we have $f(h) < f(x)$.

a) Let x be a strict local maximum for f. Prove that there is natural number $n > 0$ such that $f(h) < f(x)$ for all h such that $0 < |x - h| < 1/n$. We will say in this case that x is of size n.

b) Let x, y be two different strict local maxima for f, both of size n. Prove that $|x - y| \geq 1/n$. Conclude that the open intervals of length $1/2n$ with midpoints ot x and at y do not intersect.

c) Prove that for a given n, the set of all strict local maxima for f of size n is countable.

d) Prove that the set of all strict local maxima for f of is countable.