Math 147 - Elementary Statistics

Solutions for Spring 2020 Exam 02 Version A

This Exam is worth a total of 150 points.

Problem	1	2	3	4	5	6	7	8	9	Total
Points Earned										
Out of	18	15	15	23	20	16	20	8	15	150

1. - 18 points Answer each statement with **True** or **False**.

(I) The binomial formula:

a. can be used when drawing tickets from a box without replacement. **False**

b. gives the chance that an event occurs exactly k times out of n trials. **True**

c. can be used when the probability of success changes from trial to trial. **False**

(II) If A and B are independent events, then:

a. $P(A) = P(A|B)$. **True**

b. $P(A \textbf{ and } B) = P(A) \cdot P(B)$. **True**

c. A and B are mutually exclusive. **False**

(III) The chance that an event A happens:

a. must be between 0% and 100%. **True**

b. equals the number of times that A is expected to happen if the chance process is repeated many times. **False**

c. is equal to $1-P(\text{not } A)$, where $P(\text{not } A)$ represents the chance that A does **not** happen. **True**

2. - 15 points 500 tickets are drawn with replacement from one of the boxes below.

Box **A** = [-2 2 2 -2] Box **B** = [2 -2]

You will win two dollars if a 2 is drawn, and you will lose a dollar if a -2 is drawn.

You prefer box A **False** box B **False** neither box (it doesn't matter) **True**

Choose one of the above and **explain why!**

3. - 15 points A fair die is rolled 15 times. A player will win $\$10$ whenever a 1 is rolled and will win $\$20$ whenever a 6 is rolled. Otherwise, they will lose $\$7$.

a. Create a box model for the above random variable.

Solution to a: $\boxed{\text{1 ticket } \boxed{\$10} \text{ 1 ticket } \boxed{\$20} \text{ 4 tickets } \boxed{-\$7}}$.

b. How much does this player expect to win in total?

Solution to problem b: Average of Box $= \dfrac{10+20-7-7-7-7}{6} = \$\dfrac{1}{3}$.

There are 15 trials so the expected value is: $15 \cdot \$\frac{1}{3} = \boxed{\$\mathbf{5}}$.

4. - 23 points The winnings for a single game offered by the Golden Fortune casino have an expected value of -5 cent with an SD of one dollar (i.e., expect to lose 5 cent if you play this game). Joanne plays this game $10,000$ times.

a. Compute Joanne's total net gain. It will be negative since she loses 5 cents on average per game.

Solution: $10,000 \cdot (-0.05) = \boxed{-500}$ dollars

b. What is the SE of Joanne's total winnings?

Solution: $\sqrt{10,000} \cdot 1 = \boxed{100}$ dollars.

c. Use approximation with a normal curve to compute the probability that Joanne will lose at most 300.00 dollars. You may use the empirical rule.

Solution: $-\$300$ is 2 SEs above the average of $-\$500$ for the total winnings.

Empirical rule $\Rightarrow$ P(net loss $\geqq -\$300.00) \approx P(z \geqq 2) \approx \boxed{2.5\%}$

5. - 20 points

A box contains two red marbles and four blue marbles. Six draws are made at random with replacement from the box. Find the probability for each of the following:

a. A red marble is never drawn. $\boxed{\mathbf{(4/6)^6 \approx 0.0878}}$

b. A red marble is drawn less than two times.

$P(\text{zero draws}) \approx 0.0878$ (see part a) $P(\text{one draw}) \ = \approx 0.0878$ (see part a); $(2/6)(4/6)^5\binom{6}{1} \approx 0.2634$. Sum $\approx$ $\boxed{\mathbf{0.3512}}$

c. A blue marble is drawn exactly four times. $\boxed{\mathbf{(4/6)^4(2/6)^2\binom{6}{4} \approx 0.3292}}$

d. A blue marble is never drawn. $\boxed{\mathbf{(2/6)^6 \approx 0.0014}}$

e. A blue marble is drawn at least once. $\boxed{\mathbf{1-(2/6)^6 \approx 0.9986}}$

6. - 16 points Answer the following as precisely as possible.
a. In the following A and B are two events.
(i). State both the general multiplication rule and the general addition rule.
(ii). What is the multiplication rule in the special case that A and B are independent?
(iii). What is the addition rule in the special case that A and B are mutually exclusive?

Solution to a:

The general multiplication rule is: P(A and B) = P(A)P(B | A). If A and B are independent then the formula simplifies to P(A and B) = P(A)P(B).

The general addition rule is P(A or B) = P(A) + P(B) - P(A and B). If A and B are mutually exclusive the formula simplifies to P(A or B) = P(A) + P(B).

b. What is the binomial formula and what does it compute?

Solution to b:

$\binom{n}{k}\,p^k(1-p)^{n-k}$ It should be used when asked the probability that an event will occur exactly k times out of n trials.

7. - 20 points A die is rolled and you will win 4 dollars if the number you bet on comes up, you will lose 1 dollar if another number comes up. You play this game 36 times.

a. Compute the expected value of your total winnings.

Expected winnings per game are $4 \cdot \frac{1}{6} + (-1) \cdot \frac{5}{6} = -\frac{1}{6}$ dollars.

So for 36 games: $-36 \cdot \frac{1}{6} = \boxed{-6.00}$ dollars.

b. Compute the standard error of your total winnings.

SD per game is $|4 - (-1)| \cdot \sqrt{\frac{1}{6} \cdot \frac{5}{6}} \approx 5\sqrt{0.372678} \approx 1.863390$ dollars.

So the SE for 36 games is $\sqrt{n}\cdot$ SD of box = $\sqrt{36} \cdot 1.863390 \approx \boxed{11.18034}$ dollars.

c. Interpretation: You expect a net gain after 36 games of $\boxed{-6}$ dollars, give or take $\boxed{11.18}$ dollars or so.

8. - 8 points Fill in the blanks.

a. When A and B are **dependent** events, the probability that both A and B occur is

$\boxed{P(A) \cdot P(B|A) = P(B) \cdot P(A|B)}$

b. If a fair coin is tossed 1000 times, the **Law of Averages**. says that the number of heads will likely be close to 500, although it is unlikely that the number of heads will be exactly 500.

c. The binomial coefficient $\binom{n}{k}$ is **equal to** the binomial coefficient $\binom{n}{n-k}$. (Choose one of **less than, equal to**, or **greater than**.)

d. When drawing at random from a box of numbered tickets, the SE of the sum is the product of

√# of draws and **SD of the box**

9. - 15 points One ticket is drawn from Box **A** and one ticket is drawn from Box **B**:

Box **A** = [1 2 3 6 8] Box **B** = [1 3 6 7 8 9]

Find the probability that

a. One or both of the numbers drawn are $even$.

Solution to a:

Let U := {draw from a is even},
V := {draw from a is odd and draw from b is even}.

Then $P(U \textbf{ or } V) = P(U) + P(V)$ (addition rule) $= \frac{3}{5} + \frac{2}{5} \times \frac{2}{6} = \frac{18}{30} + \frac{4}{30} = \boxed{\mathbf{22/30 = 11/15}}$.

b. The sum of the numbers drawn is 9. Hint: Write (on scratch) all possible combinations that give a sum = 9.

Solution to b: The following table gives the different possibilities

Box a	Box b
1	8
2	7
3	6
6	3
8	1

The probability is given by: $(\frac{1}{5} \times \frac{1}{6}) \times 5 = \boxed{\mathbf{5/30 = 1/6}}$

c. One of the numbers drawn is more than twice as big as the other number. Hint: Write (on scratch) down all possible combinations that give one draw more than twice as big as the other draw.

Solution to c: The following table gives the different possibilities

Box a	Box b
1	3, 6, 7, 8, 9
2	6, 7, 8, 9
3	1, 7, 8, 9
6	1
8	1, 3

The probability is given by: $(\frac{1}{5} \times \frac{1}{6}) \times 16 = \boxed{\mathbf{16/30 = 8/15}}$