Math 148 Slides – Ch.5 – z–scores, Shifting and scaling, ...

Given is a numeric list $x_1, x_2, \dots, x_n$ of size n. Then

	average	**standard deviation**
x_j	$\bar{x}$	s_x
$y_j = x_j \pm c$	$\bar{y} = \bar{x} \pm c$	$s_y \;=\; s_x$
$u_j = x_j \cdot c \; (c \geqq 0)$	$\bar{u} = \bar{x} \cdot c$	$s_u \;=\; s_x \cdot c$
$v_j = x_j \cdot c \; (c < 0)$	$\bar{u} = \bar{x} \cdot c$	$s_v \;=\; s_x \cdot (-c)$
$w_j = x_j \cdot c + b$	$\bar{w} = \bar{x} \cdot c + b$	$s_w \;=\; s_x \cdot \lvert c \rvert$
$\tilde{x}_j = (x_j - \bar{x}) /\, s_x$	$\bar{\tilde{x}} = 0$	$s_{\tilde{x}} \;=\; 1$

In the above: **Absolute value** $\mid c \mid = \begin{cases} x & \text{if } x \geqq 0, \\ -x & \text{if } x < 0. \end{cases}$

Careful w. variances s^2: they change with the **SQUARE of the multiplier:**

If $w_j = x_j \cdot c + b$ then $s_w^2 \;=\; s_x^2 \cdot c^2$ (also for negative c! Why?)

Last line in the table: **z–scores** $\tilde{x}_j \;=\; \dfrac{x_j - \bar{x}}{s_x}$

If you can relate x_j and y_j by a shifting and scaling $y_j = a \cdot (x_j + b)$ then both lists have the same z–scores: $\boldsymbol{\tilde{y}_j \;=\; \tilde{x}_j}$