Math 148 - Formula Sheet 01

For random variables X, Y and a constant a we have

$$\begin{aligned}
\mu &= E(X) = \sum_x xP(X=x) \\
\sigma^2 &= \text{Var}(X) = \sum_x \big(x - E(X)\big)^2 P(X=X) \\
\sigma &= \text{SD}(X) = \sqrt{\text{Var}(X)} \\
E(X \pm Y) &= E(X) \pm E(Y) \\
E(aX) &= aE(X) \\
\text{Var}(X \pm Y) &= \text{Var}(X) + \text{Var}(Y) \quad \text{if } X, Y \text{ are independent}
\end{aligned}$$

Sampling distribution for the mean of iid (independendent and identically distributed) random variables $X_1, X_2, X_3, \ldots$ with mean $E(X_j) = \mu$ and SD σ.

$$\begin{aligned}
E(\bar{X}) &= E(X_1) = E(X_j) = \mu \\
\text{Var}(\bar{X}) &= \frac{\text{Var}(X_1)}{n} = \frac{\text{Var}(X_j)}{n} = \frac{\sigma^2}{n} \\
\text{SD}(\bar{X}) &= \frac{\text{SD}(X_1)}{\sqrt{n}} = \frac{\text{SD}(X_j)}{\sqrt{n}} = \frac{\sigma}{\sqrt{n}}
\end{aligned}$$

Sampling distribution formulas for proportions of iid Bernoulli trials $X_1, X_2, X_3, \ldots$ with success probability (= population proportion) p, $q = 1 - p$, and sample proportion $\hat{p}$:
If we encode success = 1 and failure = 0 then

$$\mu = E(X_1) = E(X_j) = p \text{ and } \sigma = \text{SD}(X_1) = \text{SD}(X_j) = \sqrt{pq}$$

$$\begin{aligned}
E(\hat{p}) &= E(X_1) = E(X_j) = p \\
\text{Var}(\hat{p}) &= \frac{\text{Var}(X_1)}{n} = \frac{\text{Var}(X_j)}{n} = \frac{\sigma^2}{n} = \frac{pq}{n} \\
\text{SD}(\hat{p}) &= \frac{\text{SD}(X_1)}{\sqrt{n}} = \frac{\text{SD}(X_j)}{\sqrt{n}} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{pq}{n}}
\end{aligned}$$

Geometric random variable T ("time" of the first success in an iid sequence of Bernoulli trials):

$$P(T = k) = q^{k-1}p; \quad E(T) = \mu = \frac{1}{p}; \quad \text{SD}(T) = \sigma = \sqrt{\frac{q}{p^2}}$$

Binomial random variable Y (# of successes in n iid Bernoulli trials):

$$P(Y = k) = \binom{n}{k} \cdot p^k \cdot q^{n-k} \text{ where } \binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}; \quad E(Y) = \mu = np; \quad \text{SD}(Y) = \sigma = \sqrt{npq}$$

Uniform random variable U on the interval $a \leqq x \leqq b$: If $a \leqq c \leqq d \leqq b$ then

$$P(c \leqq U \leqq d) = \frac{(d-c)}{(b-a)}; \quad E(U) = \mu = \frac{(a+b)}{2}; \quad \text{SD}(U) = \sigma = \sqrt{\frac{(b-a)^2}{12}}$$