Math 220 – Business Calculus
Spring 2013
Test 1

Name Answer Key

12 points 1. Find the domains of each function

$f(x) = 2/(4x + 6)$ $\quad 4x + 6 \neq 0 \quad x \neq -\frac{6}{4}$

$g(x) = \sqrt{6x - 3}$ $\quad 6x - 3 \geq 0 \quad x \geq \frac{3}{6}$

$y = \frac{(x-2)(x+4)}{x+4}$ $\quad x \neq -4$

$\ln((2x-6)/(x-6))$ $\quad \frac{2x-6}{x-6} > 0 \Rightarrow$ $x > 3$ & $x > 6$; $2x-6 > 0$ & $x-6 > 0$; or $2x-6 < 0$ & $x-6 < 0$; $x < 3$ & $x < 6$ $\Rightarrow$ $x > 6$ or $x < 3$

18 pts 2. Solve each equation for x

$e^{3x^2+5x} = e^{-2}$ $\quad 3x^2 + 5x = -2$

$3x^2 + 5x + 2 = (3x + 2)(x + 1) = 0$ $\quad x = -1$ or $x = -2/3$

$4^{3x+2} = 64$

$= 4^3$ $\quad 3x + 2 = 3$

$x = 1/3$

$3^x = 6(4^x)$

$\ln 3^x = x \ln 3 = \ln(6)(4^x) = \ln 6 + x \ln 4$

$x = \frac{\ln 6}{\ln 3 - \ln 4}$

$\ln(4x - 3) = 8$

$4x - 3 = e^8$ $\quad x = \frac{e^8 + 3}{4}$

$\log_3(x - 4) = 2$

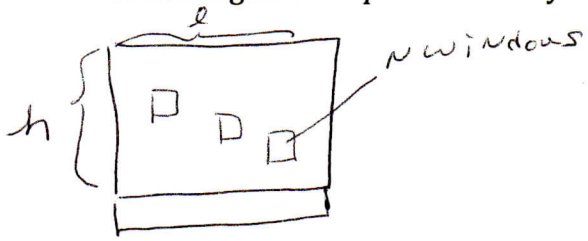
$x - 4 = 3^2 = 9$ $\quad x = 13$

$\log_2(x - 3) - \log_2(x + 2) = \log_2 7$

$\log_2\left(\frac{x-3}{x+2}\right) = \log_2 7$ $\quad \frac{x-3}{x+2} = 7$

$x - 3 = 7x + 14$ $\quad x = -\frac{17}{6}$

12 pts 3. A wall of a room is to be painted. Some windows are also to be installed in the wall and a baseboard is to be nailed along the bottom of the wall. Paint costs $3 per square meter. Windows are 0.6 square meters each and cost $100. Baseboards cost $12 per meter. Write an equation for the cost of finishing the wall as a function of the length (l) and height (h) of the wall and the number of windows (n). (Hint: pick some numbers for the length, height and number of windows. Find the cost using those numbers and then use the same method to find a general equation for any values of length, height and number of windows)



Area to paint: size of wall − size of windows
$= lh - .6n$

Cost: paint + windows + baseboard

$$C(l, h, n) = 3(lh - .6n) + 100n + 12l$$

3 pts 4. A couple has received an inheritance and wants to invest the money for their child's college education. They can invest the money at 5% paid continuously. How much should they invest in order to have $120,000 in 10 years? (DO NOT simplify your answer.)

$$PV = \frac{120{,}000}{e^{(.05)(10)}} \quad \text{or} \quad 120{,}000\, e^{-.05(10)}$$

5 pts 5. An investor puts $4000 into a long term investment that pays 4% continuously). At the end of 25 years will the investor have earned more or less than $10,000? Justify your answer.

$$FV = 4000\, e^{(.04)(25)} = 4000\, e^{1}$$

Since $10{,}000 = 4000(2.5)$ and $e \approx 2.7 > 2.5$

The investor will have more than $10,000

5 pts 6. What is the effective interest rate of 8% compounded semi-annually? (DO simplify your answer)

$$r_{eff} = \left(1 + \frac{.08}{2}\right)^2 - 1 = 1.0816 - 1 = .0816$$

or 8.16%

1.04
1.04
4 16
1 04
1.0816

20 pts 7. Tell whether this function is continuous or discontinuous at each of the points listed. Explain your answer using the formal definition of continuity.

$$f(x)=\begin{cases}2x+5 & x<1\\ 8-x & 1<x<4\\ 3x-7 & 4\le x\le 6\\ 2x-1 & x>6\end{cases}$$

2 a) x = 1 $f(1)$ DNE so discontinuous

6 b) x = 3 $8-x$ is a polynomial and x is in the interval $(1,4)$ $\therefore$ $f(x)$ is continuous

6 c) x = 4 $\lim_{x\to 4^-} 8-x = 4$ $\lim_{x\to 4^+} 3x-7 = 5$ $4 \neq 5$ discontinuous

6 d) x = 6 $\lim_{x\to 6^-} 3x-7 = 11$ $\lim_{x\to 6^+} 2x-1 = 11$ $f(6)=3(6)-7 = 11$ continuous

25 pts 8. Find each limit (Show some work)

$$\lim_{x\to 3^+}\frac{x^2+2x-15}{x-3} \quad (\to 0/0) = \lim_{x\to 3^+}\frac{(x+3)(x+5)}{(x-3)} = 3+5 = 8$$

$$\lim_{x\to -4^-}\frac{2x^2-x-8}{x+4} \quad (\to 28/0^-) = -\infty$$

$$\lim_{x\to 25}\frac{25-x}{5-\sqrt{x}}\cdot\frac{(5+\sqrt{x})}{(5+\sqrt{x})} = \lim_{x\to 25}\frac{(25-x)(5+\sqrt{x})}{25-x} = 10$$

$$\lim_{x\to\infty}(\ln(4x-5)-\ln(4x^2+2)) = \lim_{x\to\infty}\ln\left(\frac{4x-5}{4x^2+2}\right) \quad \left(\frac{4x-5}{4x^2+2}\to 0\right) = -\infty$$

$$\lim_{x\to\infty} e^{x^3+2x^2-5} \quad (x^3+2x^2-5\to\infty) = \infty$$

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12 points 1. Find the domains of each function

$f(x) = 4/(3x - 6)$ $\quad 3x - 6 \neq 0 \quad x \neq 2$

$g(x) = \sqrt{6x + 4}$ $\quad 6x + 4 \geq 0 \quad x \geq -\frac{4}{6}$

$y = \frac{(x-2)(x+4)}{x-2}$ $\quad x \neq 2$

$\ln((2x-4)/(x+8))$ $\quad x \neq -8$ and $\frac{2x-4}{x+8} > 0 \Rightarrow 2x-4>0$ & $x+8>0$ or $2x-4<0$ & $x+8<0$

$x > 2$ or $x < -8$ $\quad \Leftarrow \quad x<2$ & $x<-8$

18 points 2. Solve each equation for x

$e^{3x^2-8x} = e^3$ $\quad 3x^2-8x=3$ $\quad 3x^2-8x-3=0$ $\quad (3x+1)(x-3)=0$ $\quad x = -1/3$ or $x = 3$

$5^{4x+1} = 125 = 5^3$ $\quad 4x+1=3 \quad x = \frac{1}{2}$

$2^x = 3(6^x)$

$\ln(2^x) = \ln(3(6^x)) = \ln 3 + \ln 6^x$

$x \ln 2 = \ln 3 + x \ln 6 \quad x = \frac{\ln 3}{\ln 2 - \ln 6}$

$\ln(6x + 5) = 2$

$6x+5 = e^2$

$x = (e^2 - 5)/6$

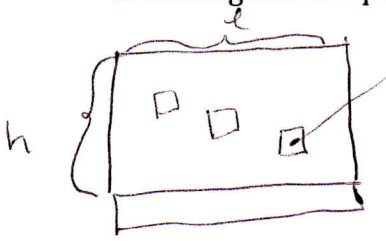
$\log_4(x + 5) = 2$

$x+5 = 4^2 = 16 \quad x = 11$

$\log_4(x + 5) - \log_4(x - 7) = \log_4 5$

$\log_4\left(\frac{x+5}{x-7}\right) = \log_4 5 \quad \frac{x+5}{x-7} = 5 \quad x+5 = 5x-35 \quad x = 10$

12 points 3. A wall of a room is to be painted. Some windows are also to be installed in the wall and a baseboard is to be nailed along the bottom of the wall. Paint costs $2 per square meter. Windows are 0.8 square meters each and cost $200. Baseboards cost $15 per meter. Write an equation for the cost of finishing the wall as a function of the length (l) and height (h) of the wall and the number of windows (n). (Hint: pick some numbers for the length, height and number of windows. Find the cost using those numbers and then use the same method to find a general equation for any values of length, height and number of windows)



Area to paint: size of wall − size of windows

$= (lh - .8n)$ sq. meters

Cost: paint + windows + baseboard

$C(l,h,n) = 2(lh - .8n) + 200n + 15l$

3 points 4. A couple has received an inheritance and wants to invest the money for their child's college education. They can invest the money at 3% paid continuously. How much should they invest in order to have $80,000 in 12 years? (DO NOT simplify your answer.)

$PV = \frac{80,000}{e^{(.03)(12)}}$ or $80,000\, e^{-.03(12)}$

5 pts 5. An investor puts $4000 into a long term investment that pays 4% continuously). At the end of 25 years will the investor have earned more or less than $12,000? Justify your answer.

$FV = 4000\, e^{(.04)(25)} = 4000\, e^{1}$

Since $12000 = 4000(3)$ and $e^1 \approx 2.7 < 3$

The investor will have less than $12,000

5 pts 6. What is the effective interest rate of 4% compounded semi-annually? (DO simplify your answer)

$r_{effective} = \left(1 + \frac{.04}{2}\right)^2 - 1 = 1.0204 - 1 = .0204$

or 4.04%

$$\begin{array}{r} 1.02 \\ \times\ 1.02 \\ \hline 204 \\ 102 \\ \hline 1.0404 \end{array}$$

20 pts 7. Tell whether this function is continuous or discontinuous at each of the points listed. Explain your answer using the formal definition of continuity.

$$f(x)=\begin{cases}2x+5 & x\le 1\\ 8-x & 1<x<4\\ 3x-8 & 4<x\le 6\\ 2x+2 & x>6\end{cases}$$

6 a) x = 1 $\lim_{x\to 1^-} 2x+5 = 7$ $\lim_{x\to 1^+} 8-x = 7$ $f(1) = 2(1)+5 = 7$

continuous

2 b) x = 4 $f(4)$ DNE discontinuous

6 c) x = 5 $3x-8$ is a polynomial 5 is in the open interval $(4,6)$ ∴ continuous

6 d) x = 6 $\lim_{x\to 6^-} 3x-8 = 10$ $\lim_{x\to 6^+} 2x+2 = 14$ $10 \neq 14$

discontinuous

25 pts 8. Find each limit (Show some work)

$$\lim_{x\to -5^-}\frac{x^2+2x-15}{x+5} \quad (\to 0) = \lim_{x\to -5^-}\frac{(x-3)(x+5)}{(x+5)} = -5-3 = -8$$

$$\lim_{x\to -4^+}\frac{2x^2-x-8}{x+4} \quad (\to 28 \text{ over } \to 0^+) = +\infty$$

$$\lim_{x\to 16}\frac{16-x}{4-\sqrt{x}} \quad (\to 0) = \lim_{x\to 16}\frac{(16-x)(4+\sqrt{x})}{(16-x)} = 8$$

$$\lim_{x\to\infty}(\ln(3x^2-2)-\ln(3x+4)) = \lim_{x\to\infty}\ln\left(\frac{3x^2-2}{3x+4}\right) \quad \left(\frac{3x^2-2}{3x+4}\right)\to\infty \Rightarrow \ln(\infty) = \infty$$

$$\lim_{x\to -\infty} e^{x^3+2x^2-5} \quad (x^3+2x^2-5 \to -\infty) = 0$$