Math 220 – Business Calculus
Spring 2013
Test 2

Name ____________________

1. Find the indicated derivative of each function. If necessary, show your work on the extra paper provided.

a) $f(x) = x^5 + \frac{2}{x^2} + \frac{4}{x} + 6x + \sqrt[3]{x}$ f'(x)= $5x^4 - \frac{4}{x^3} - \frac{4}{x^2} + 6 + \frac{1}{3}x^{-2/3}$

b) $C(q) = \ln(5q^3 - 3q)(q^4 + 4q + 7)$ dC/dq =

$\ln(5q^3-3q)(4q^3+4) + \frac{15q-3}{5q^3-3q}(q^4+4q+7)$

c) h(t) = ln(t^3) dh/dt = $3t^2/t^3$

d) $s(y) = \log_3 y$ ds/dy = $\frac{1}{y \ln 3}$

e) $y = \left(\frac{3x+4}{5x-2}\right)^{12}$ y' = $12\left(\frac{3x+4}{5x-2}\right)^{11}\left(\frac{(5x-2)(3)-(5)(3x+4)}{(5x-2)^2}\right)$

f) $q(p) = \sqrt[3]{7{,}000 + p^5}$ dq/dp = $\frac{1}{3}(7000+p^5)^{-2/3}(5p^4)$

g) r(w) = 5^w dr/dw $= 5^w \ln 5$

h) $s(x) = e^{\sqrt{3x^5+2x-8}}$ s'(x) $\left(e^{\sqrt{3x^5+2x-8}}\right)\left(\frac{1}{2}(3x^5+2x-8)^{-1/2}\right)(15x^4+2)$

i) $v(t) = \frac{1}{e^{4t}} = e^{-4t}$ dv/dt = $-4e^{-4t}$

j) $h(w) = 2w^4 + 4w^2 - 2w + 5$ h''(w) = $24w^2 + 8$

$h'(w) = 8w^3 + 8w - 2$

k) $z(k) = e^{(4k^2)}$ d²z/dk² = $(e^{4k^2})(8) + (e^{4k^2})(8k)(8k)$

$\frac{dz}{dk} = (e^{4k^2})(8k)$

2. Find the equation for the line tangent to the curve $f(x) = 2x^3 + 3x^2 - 5x + 4$ at $x = 2$. Leave the equation in point-slope form.

$f'(x) = 6x^2 + 6x - 5$

$m = f'(2) = 6(4) + 6(2) - 5 = 31$

$y = f(2) = 2(8) + 3(4) - 5(2) + 4 = 22$

$y - 22 = 31(x - 2)$

3. The total sales (P) in thousands of dollars made on a popular product can be expressed as a function of the number of weeks (w) since the product first went on the market according to this equation: $P(w) = 30\sqrt{w} + 4w$

Be sure to include the proper units in your answers

a) What is the average rate of increase in sales from the end of week 1 ($w = 1$) to the end of week 4 ($w = 4$)?

$$\text{Avg} = \frac{P(4) - P(1)}{4 - 1} = \frac{(30\sqrt{4} + 4(4)) - (30\sqrt{1} + 4(1))}{3}$$

$$= \frac{76 - 34}{3} = 14 \text{ ~~sales~~ Thousand \$/week}$$

b) What is the rate of increase of sales at the end of week 9?

$$P'(w) = \frac{30}{2\sqrt{w}} + 4$$

$$P'(9) = \frac{30}{2\sqrt{9}} + 4 = 9 \text{ ~~sales~~ Thousand \$/week}$$

4. The profit made on the sale of a product is a function of the quantity sold given by the function $P(q) = 4.4q - .01q^2$.

a) Write an equation for the marginal profit as a function of the quantity produced.

$P'(q) = 4.4 + .02q$

b) What profit will be made by selling 100 items?

$P(100) = 440 - 100 = \$340$

c) Find an estimate, using marginal profit, for the increase in profit if the number of items sold increases from 100 to 101. **<u>DO NOT calculate the actual cost to produce 101 items.</u>**

$P'(100) = 4.4 - .02(100) = \$2.40$

5. A piece of machinery is moving along a track. Its distance from the starting point (in meters) is given by the equation $s = t^2 - 6t$ where t is the time measured in seconds. A positive distance means the machine is to the right of the starting point. A negative distance means that it is to the left of the starting position.

a) How far from the starting point is the machine after 3 seconds? In which direction?

$s(3) = 9 - 6(3) = -9$ meters

9 meters left

b) How fast is the machine moving after 6 seconds? Is it moving left or right?

$v(t) = 2t - 6$

$v(6) = 12 - 6 = 6$ m/sec moving right

c) When will the machine be 27 meters to the right of the starting position?

$t^2 - 6t = 27$

$(t-9)(t+3) = 0$

$t^2 - 6t - 27 = 0$

$t = 9$ seconds

d) When will the machine be moving left at 4 meters/second?

$2t - 6 = -4$

$2t - 2 = 0$

$t = 1$ second

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1. Find the indicated derivative of each function. If necessary, show your work on the extra paper provided.

a) $f(x) = x^4 + \frac{5}{x} + \frac{2}{x^2} + 4x + \sqrt[4]{x}$ f'(x)= $4x^3 - \frac{5}{x^2} - \frac{4}{x^3} + 4 + \frac{1}{4}x^{-3/4}$

b) $C(q) = \ln(5q^5 + 4q)(q^3 - 3q + 5)$ dC/dq =

$\ln(5q^5+4q)(3q^2-3) + \frac{25q^4+4}{5q^5+4q}(q^3-3q+5)$

c) $h(m) = \ln(m^4)$ dh/dm = $\frac{4m^3}{m^4}$

d) $s(t) = \log_5 t$ ds/dt = $\frac{1}{t \ln 5}$

e) $g(x) = \left(\frac{5x+2}{4x-6}\right)^{18}$ g '(x) = $18\left(\frac{5x+2}{4x-6}\right)^{17}\left(\frac{(4x-6)5 - 4(5x+2)}{(4x-6)^2}\right)$

f) $q(p) = \sqrt[4]{5{,}000 + p^3}$ dq/dp = $\frac{1}{4}(5000+p^3)^{-3/4}(3p^2)$

g) $r(w) = 6^w$ dr/dw = $6^w \ln 6$

h) $s(x) = e^{\sqrt{3x^4+2x^2+9}}$ s'(x) $= \left(e^{\sqrt{3x^4+2x^2+9}}\right)\left(\frac{1}{2}(3x^4+2x^2+9)^{-1/2}\right)(12x^3+4x)$

i) $v(t) = \frac{1}{e^{6t}} = e^{-6t}$ dv/dt = $-6e^{-6t}$

j) $h(k) = 6k^3 + 5k^2 - 7k + 3$ h"(k) = $36k + 10$

$h'(k) = 18k^2 + 10k - 7$

k) $z(w) = e^{(3w^2)}$ $d^2z/dw^2 = (e^{3w^2})(6) + (e^{3w^2})(6w)(6w)$

$\frac{dz}{dw} = (e^{3w^2})(6w)$

2. Find the equation for the line tangent to the curve $f(x) = x^3 + 4x^2 - 6x + 3$ at $x = 2$. Leave the equation in point-slope form.

$$f'(x) = 3x^2 + 8x - 6$$

$$m = f'(2) = 3(4) + 8(2) - 6 = \cancel{28}\ 22$$

$$y = f(2) = 8 + 4(4) - 6(2) + 3 = 15$$

$$y - 15 = \overset{22}{\cancel{28}}(x - 2)$$

3. The total sales (P) in thousands of dollars made on a popular product can be expressed as a function of the number of weeks (w) since the product first went on the market according to this equation: $P(w) = 20\sqrt{w} + 5w$

Be sure to include the proper units in your answers

a) What is the average rate of increase in sales from the end of week 1 (w = 1) to the end of week 4 (w = 4)?

$$Avg = \frac{P(4) - P(1)}{4 - 1} = \frac{(20\sqrt{4} + 5(4)) - (20\sqrt{1} + 5(1))}{3}$$

$$= \frac{60 - 25}{3} = \frac{35}{3}\ \cancel{\text{sales}}/\cancel{\text{week}}$$

$$= \$11,667/\text{week}$$

b) What is the rate of increase of sales at the end of week 9?

$$P'(w) = \frac{20}{2\sqrt{w}} + 5$$

$$P'(9) = \frac{20}{2\sqrt{9}} + 5 = \frac{25}{3}\ \cancel{\text{sales}}/\cancel{\text{week}}$$

$$= \$8,333/\text{week}$$

4. The profit made on the sale of a product is a function of the quantity sold given by the function $P(q) = 5.7q - .01q^2$.

a) Write an equation for the marginal profit as a function of the quantity produced.

$P'(q) = 5.7 - .02q$

b) What profit will be made by selling 100 items?

$P(100) = 570 - 100 = \$470$

c) Find an estimate, using marginal profit, for the increase in profit if the number of items sold increases from 100 to 101. **<u>DO NOT calculate the actual cost to produce 101 items.</u>**

$P'(100) = 5.7 - .02(100) = \$3.70$

5. A piece of machinery is moving along a track. Its distance from the starting point (in meters) is given by the equation $s = t^2 - 7t$ where t is the time measured in seconds. A positive distance means the machine is to the right of the starting point. A negative distance means that it is to the left of the starting position.

a) How far from the starting point is the machine after 4 seconds? In which direction?

$S(4) = 16 - 28 = -12$ meters

12 meters To the left

b) How fast is the machine moving after 8 seconds? Is it moving left or right?

$V(t) = 2t - 7$

$V(8) = 16 - 7 = 9$ m/second to the right

c) When will the machine be 30 meters to the right of the starting position?

$t^2 - 7t = 30$ $(t - 10)(t + 3) = 0$

$t^2 - 7t - 30 = 0$ $t = 10$ seconds

d) When will the machine be moving left at 5 meters/second?

$2t - 7 = -5$

$2t - 2 = 0$

$t = 1$ second