Math 220 – Business Calculus
Spring 2013
Test 3 (180 points)

Name Answer Key

14 PTS 1. Find these limits

$$\lim_{x\to -5^-} \frac{x^2+2x-15}{x+5} = \lim_{x\to -5^-} \frac{(x+5)(x-3)}{\cancel{(x+5)}} = -8$$

$$\lim_{x\to -\infty} \frac{x^2+2x-15}{x+5} = \lim_{x\to -\infty} \frac{x^2}{x} = \lim_{x\to -\infty} x = -\infty$$

7 PTS 2. Find dy/dx for this curve

$5x^2 + 7y^2 = xy$

$$10x + 14y\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\frac{dy}{dx} = \frac{y-10x}{14y-x}$$

20 pts 3. Find the equation for the line tangent to this curve at the given point

$x^2y^3 + 6y = 4y^2x^2 - 42$ at the point (4,1)

$$2xy^3 + x^2(3y^2)\frac{dy}{dx} + 6\frac{dy}{dx} = \left(8y\frac{dy}{dx}\right)x^2 + 4y^2 2x$$

$$\frac{dy}{dx} = \frac{8xy^2 - 2xy^3}{3x^2y^2 + 6 - 8x^2y}$$

$$\left.\frac{dy}{dx}\right|_{(4,1)} = \frac{8(4)(1) - 2(4)(1)}{3(16)1 + 6 - 8(16)(1)} = -\frac{24}{74}$$

$$y - 1 = -\frac{24}{74}(x-4)$$

15pts 4. The demand (q) for a product drops as the price (p) increases according to this relationship:

$$q = 5000 - 36\sqrt[4]{p}$$

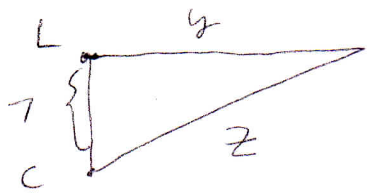
How fast is demand changing if the price is increasing at a rate of \$3.00 per month when the price is \$16.00

$$\frac{dq}{dt} = -36\left(\frac{1}{4}\right)p^{-3/4}\frac{dp}{dt}$$

$$= \frac{-9(3)}{\left(\sqrt[4]{16}\right)^3} = -\frac{27}{8} \text{ units/mo}$$

$$= -3\,3/8 \text{ units/mo}$$

30pts 5. Leslie is in the middle of a 14 m wide race track. Chris is on the side of the track 7 meters away from Leslie. Leslie starts running at 6m/sec. How fast is Leslie moving away from Chris when Leslie is 20m from the starting point? (You may leave your answer in radical form)



$$z^2 = y^2 + 49$$

$$\cancel{2}z\frac{dz}{dt} = \cancel{2}y\frac{dy}{dt}$$

$y = 20$

$z^2 = 20^2 + 49$

$= 449$

$z = \sqrt{449}$

$$(\sqrt{449})\frac{dz}{dt} = 20(6)$$

$$\frac{dz}{dt} = \frac{120}{\sqrt{449}} \text{ m/sec}$$

42 PTS 6. Use this information about a function to answer the questions and graph the function on the graph paper you have been given.

$$f(x) = \frac{x^2}{2} - \frac{8}{x} \qquad f'(x) = x + \frac{8}{x^2} \qquad f''(x) = 1 - \frac{16}{x^3}$$

a. What is the domain of f(x)?

$\mathbb{R}\ \ x \neq 0$

b. Give the equations of any asymptotes of the function.

$\lim_{x\to\infty} f(x) = \infty \qquad \lim_{x\to-\infty} f(x) = \infty$ No Horiz. Asymp.

$\lim_{x\to 0^-} f(x) = \infty \qquad \lim_{x\to 0^+} f(x) = -\infty$ Vert. Asymp at $x = 0$

c. Find the x and y-intercepts (if any)

$f(0)$ DNE no y-intercept

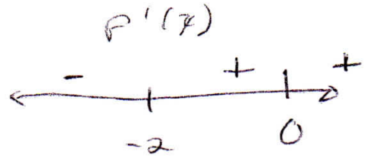
$\frac{x^2}{2} - \frac{8}{x} = 0 \qquad \frac{x^3-16}{2x} = 0 \qquad x = \sqrt[3]{16} = 2\sqrt[3]{2}$ x-intercept

d. Find the x and y coordinates of all the critical points

$f'(x) = x + \frac{8}{x^2} = 0$

$\frac{x^3+8}{x^2} = 0$ at $x = \sqrt[3]{-8}$

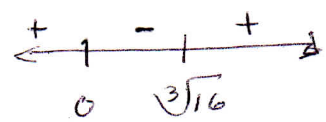
$x = -2$



$f(-2) = \frac{4}{2} - \frac{8}{-2} = 6 \qquad \boxed{(-2, 6)\ CP}$

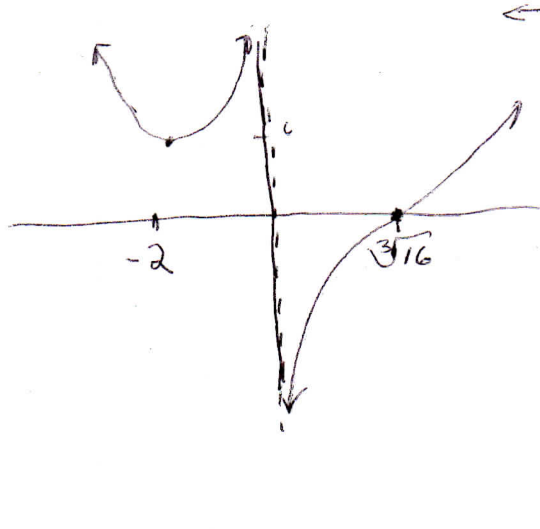
e. Find the x and y coordinates of all the inflection points

$f''(x) = 1 - \frac{16}{x^3} = 0 \qquad \frac{x^3-16}{x^3} = 0 \qquad x = \sqrt[3]{16} = 2\sqrt[3]{2}$



$\boxed{IP\ (\sqrt[3]{16}, 0)}$

graph 10 PTS



42 pts 7. Use the graph of the function f below to identify the items below as closely as you can. **If some item does not exist, write "NONE."**

a) The domain of the function

$\mathbb{R}$ $x \neq -2$ & $x \neq 4$

b) Equations of any asymptotes

$x = -2$, $x = 4$

c) The interval(s) where the function is increasing and where it is decreasing

inc $(-5, -2)$ $(-2, 4)$ $(4, 8)$ dec $(-\infty, -5)$ $(8, \infty)$

d) The interval(s) where the function is concave up and where it is concave down

up $(-\infty, -2)$ $(1, 4)$ down $(-2, 1)$ $(4, \infty)$

e) The (x,y) coordinate(s) of critical points

$(-5, -1)$ ~~(8, 2)~~ ~~(0, 4)~~ ~~(5.5, 9.5~~

$(8, 2)$

f) The (x,y) coordinates of any points of inflection

$(1, 1)$ ~~(0, 4)~~ ~~(5.5, 9~~

g) The intervals where f(x) > 0

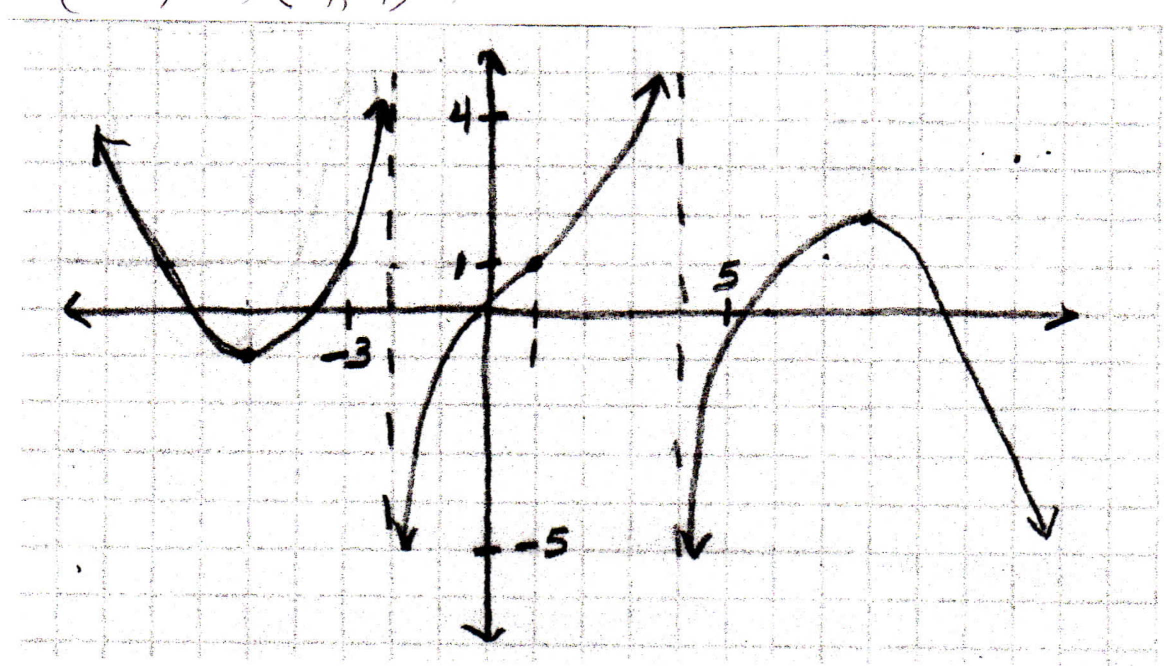
$(-\infty, -6)$ $(-4, -2)$ $(0, 4)$, $(5.5, 9.5)$

h) The intervals where f'(x) > 0

$(-5, -2)$ $(-2, 4)$ $(4, 8)$

i) The intervals where f "(x) > 0

$(-\infty, -2)$ $(1, 4)$



Math 220 – Business Calculus
Spring 2013
Test 3 (180 points)

Name Answer Key

14 pts 1. Find these limits

$$\lim_{x\to 3^+} \frac{x^2+2x-15}{x-3} = \lim_{x\to 3^+} \frac{\cancel{(x-3)}(x+5)}{\cancel{(x-3)}} = 8$$

$$\lim_{x\to\infty} \frac{x^2+2x-15}{x-3} = \lim_{x\to\infty} \frac{x^2}{x} = \lim_{x\to\infty} x = \infty$$

7 pts 2. Find dy/dx for this curve

$4x^2 + 5y^2 = xy$

$$8x + 10y\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\frac{dy}{dx} = \frac{y-8x}{10y-x}$$

20 pts 3. Find the equation for the line tangent to this curve at the given point

$x^2y^2 + 3x = 4y^3x + 6$ at the point (3,1)

$$x^2 2y\frac{dy}{dx} + 2xy^2 + 3 = 4y^3 + 12y^2\frac{dy}{dx}x$$

$$\frac{dy}{dx} = \frac{4y^3 - 2xy^2 - 3}{x^2 2y - 12xy^2}$$

$$\left.\frac{dy}{dx}\right|_{(3,1)} = \frac{4(1) - 2(3)(1) - 3}{(9)(2)(1) - 12(3)(1)} = \frac{5}{18}$$

$$y - 1 = \frac{5}{18}(x-3)$$

15 pts 4. The demand (q) for a product drops as the price (p) increases according to this relationship:

$$q = 7000 - 12\sqrt[3]{p}$$

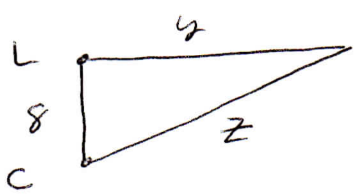
How fast is demand changing if the price is increasing at a rate of \$6.00 per month when the price is \$27.00

$$\frac{dq}{dt} = (-12)\left(\frac{1}{3}\right)p^{-2/3}\frac{dp}{dt}$$

$$= \frac{-4(6)}{(\sqrt[3]{27})^2} = -\frac{24}{9} \text{ units/mo}$$

$$= -2\tfrac{6}{9} \text{ units/mo}$$

30 pts 5. Leslie is in the middle of a 16 m wide race track. Chris is on the side of the track 8 meters away from Leslie. Leslie starts running at 5m/sec. How fast is Leslie moving away from Chris when Leslie is 15m from the starting point? (You may leave your answer in radical form)



$$z^2 = y^2 + 64$$

$$\cancel{2}z\frac{dz}{dt} = \cancel{2}y\frac{dy}{dt}$$

$$y = 15$$

$$z^2 = 15^2 + 64$$

$$= 289$$

$$z = \sqrt{289}$$

$$(\sqrt{289})\frac{dz}{dt} = 15(5)$$

$$\frac{dz}{dt} = \frac{75}{\sqrt{289}} \text{ m/sec}$$

42 pts 6. Use this information about a function to answer the questions and graph the function on the graph paper you have been given.

$f(x) = x^2 - \frac{2}{x}$ $\quad f'(x) = 2x + \frac{2}{x^2}$ $\quad f''(x) = 2 - \frac{4}{x^3}$

a. What is the domain of f(x)?

$\mathbb{R}$ $x \neq 0$

b. Give the equations of any asymptotes of the function.

$\lim_{x \to \infty} f(x) = \infty$ $\quad \lim_{x \to -\infty} f(x) = \infty$ $\quad$ No Horiz. Assmp.

$\lim_{x \to 0^-} f(x) = \infty$ $\quad \lim_{x \to 0^+} f(x) = -\infty$ $\quad$ VA at $x = 0$

c. Find the x and y-intercepts (if any)

$f(0)$ DNE $\quad$ No y-intercept

$x^2 - \frac{2}{x} = \frac{x^3 - 2}{x} = 0$ at $x = \sqrt[3]{2}$ $\quad$ x intercept

d. Find the x and y coordinates of all the critical points

$2x + \frac{2}{x^2} = \frac{2x^3 + 2}{x^2} = 0$ at $x = -1$

$y = (-1)^2 - \frac{2}{-1} = 3$

$f'(x)$ $\quad$ $-$ $\quad$ -1 $\quad$ $+$ $\quad$ 0 $\quad$ $+$

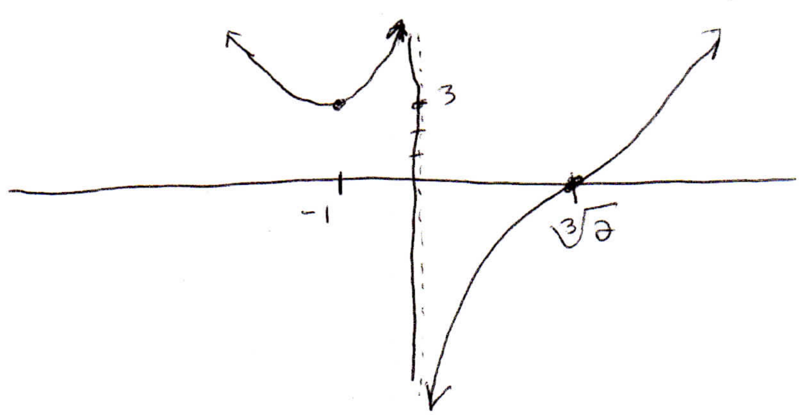
5. Find the x and y coordinates of all the inflection points $\quad 2 - \frac{4}{x^3} = \frac{2x^3 - 4}{x^3} = 0$

$f''(x)$ $\quad$ $+$ $\quad$ 0 $\quad$ $-$ $\quad$ $\sqrt[3]{2}$ $\quad$ $+$

at $x = \sqrt[3]{2}$

$y = 0$ (See part c)

Graph 10 pts



42 pts

7. Use the graph of the function f below to identify the items below as closely as you can. **<u>If some item does not exist, write "NONE."</u>**

a) The domain of the function

$\mathbb{R}$. $x \neq -4$ or 2

b) Equations of any asymptotes

$x = -4$ $\quad x = 2$

c) The interval(s) where the function is increasing and where it is decreasing

inc $(-\infty, -7)$ $(5, \infty)$ dec $(-7, -4)$ $(-4, 2)$ $(2, 5)$

d) The interval(s) where the function is concave up and where it is concave down

up $(-4, -1)$ $(2, \infty)$ down $(-\infty, -4)$ $(-1, 2)$

e) The (x,y) coordinate(s) of critical points

$(-7, -1)$ $(5, -2)$

f) The (x,y) coordinates of any points of inflection

$(-1, -1)$

g) The intervals where f(x) > 0

$(-4, -2)$ $(2, 3)$ $(6.5, \infty)$

h) The intervals where f'(x) > 0

$(-\infty, -7)$ $(5, \infty)$ same as c) inc

i) The intervals where f "(x) > 0

$(-4, -1)$ $(2, \infty)$ same as d) up