Math 220 – Business Calculus
Spring 2013
Test 4 (140 points)

Name Answer Key
Version A

20 1. A specialty grocery store can sell 400 quarts of pure maple syrup at $21.00 per quart. For each drop in price of $0.10 two more quarts can be sold. What price should the store charge for the maple syrup to maximize revenue?

$p = 21 - .1x \quad q = 400 + 2x \quad R = (21 - .1x)(400 + 2x)$

$R = 8400 - 40x + 42x - .2x^2 = 8400 + 2x - .2x^2$

$R' = 2 - .4x = 0 \quad x = 5$

$x$ = # of price changes.

$p = 21 - (.1)(5) = \$20.50$

2. The quantity of a product that can be sold varies with the price according to this demand function: $q = 1440 - 0.3p^2$

5 a) What is the general formula for elasticity? $E(p) = -\frac{p}{q}\frac{dq}{dp}$

5 b) What is the equation for elasticity as a function of price for this problem?

$E(p) = \left(\frac{-p}{1440 - .3p^2}\right)(-.6p) = \frac{+.6p^2}{1440 - .3p^2}$

7 c) Use elasticity to find the price that will yield the highest revenue. No other solution will be accepted.

$\frac{.6p^2}{1440 - .3p^2} = 1$

$.6p^2 = 1440 - .3p^2$

$.9p^2 = 1440$

$p^2 = \frac{1440}{.9} = \frac{14,400}{9} =$

$p^2 = 1600$

$p = \$40$

3 d) At a price of $45 is the demand elastic or inelastic?

elastic

3. Evaluate each integral.

Definite integrals should have a numerical expression for an answer. The expression does not need to be simplified except for part c. For example: $(3^2/2) - (1^2/2)$ is a sufficient answer.

Indefinite integrals should satisfy boundary conditions <u>if</u> any are given.

6 a) $\int \left(5x^2 + 2\sqrt[5]{x} + \frac{7}{x^4} + \frac{3+x}{\sqrt{x}}\right) dx$ $= \frac{5x^3}{3} + \frac{2x^{4/5}}{4/5} + \frac{7x^{-3}}{-3}$

$+ \frac{3x^{1/2}}{1/2} + \frac{x^{3/2}}{3/2} + C$

3 b) $\int_{-1}^{5} 5e^{3x}\, dx$ $= \frac{5e^{3x}}{3}\Big|_{-1}^{5} = \frac{5e^{15}}{3} - \frac{5e^{-3}}{3}$

5 c) $\int_{1}^{e^3} \frac{2}{x} dx$ Note: you must simplify this problem down to a single number.

$= 2\ln|x|\Big|_{1}^{e^3} = 2\ln e^3 - 2\ln 1 = 6$

6 d) $\int_{3}^{6} 6x^3 \ln x\, dx$ $= \frac{6x^4}{4}\ln x\Big|_3^6 - \int_3^6 \frac{6x^4}{4}\cdot\frac{1}{x}\,dx$

$dv \quad u$

$v = \frac{6x^4}{4} \quad du = \frac{1}{x}dx$

$= \left(\frac{6x^4}{4}\ln x - \frac{6x^4}{16}\right)\Big|_3^6$

$= \left(\frac{6(6)^4}{4}\ln 6 - \frac{6(6)^4}{16}\right) - \left(\frac{6(3)^4}{4}\ln 3 - \frac{6(3)^4}{16}\right)$

3. continued

6 e) $\int_0^2 (9x^2+3)/\sqrt{2x^3+2x+1}dx$ $= \frac{3}{2}\int_1^{21} \frac{1}{\sqrt{u}}\,du = \frac{3}{2}\left(\frac{u^{1/2}}{1/2}\right)\Big|_1^{21}$

$u = 2x^3+2x+1$

$\frac{du}{dx} = 6x^2+2$

$= 3\left(\sqrt{21}-\sqrt{1}\right)$

$dx = \frac{du}{6x^2+2}$

$2(2)^3+2(2)+1 = 21$

4 f) $\int 5x\sqrt{3+x}dx$ $= \int 5(u-3)\sqrt{u}\,du = \int\left(5u^{3/2}-15u^{1/2}\right)du$

$u = 3+x$ $\quad x = u-3$

$\frac{du}{dx} = 1$ $\quad = \frac{5u^{5/2}}{5/2} - \frac{15u^{3/2}}{3/2} = \frac{5(3+x)^{5/2}}{5/2} - \frac{15(3+x)^{3/2}}{3/2}$

4. Someone is standing on a cliff 200 m above the valley below. That person throws a rock downward with an initial velocity of 20 m sec. The acceleration due to gravity is 9.8 m/sec$^2$. (Remember that downward velocity and acceleration are both negative.) How far above the ground will the rock be after 2 seconds?

$V(t) = \int -9.8\,dt = -9.8t + C$ $\quad V(0) = -20 \Rightarrow C = -20$

$P(t) = \int(-9.8t-20)\,dt = 4.9t^2 - 20t + C$

$P(0) = 200 \Rightarrow C = 200$

$P(t) = -4.9t^2 - 20t + 200$

$P(2) = -4.9(4) - 20(2) + 200$

$= -19.6 - 40 + 200 =$ ~~139.6m~~ 140.4 m

20

5. Find the area between these two functions when $1 \le x \le 6$: Note: Once you have a numerical answer DO NOT SIMPLIFY! Be sure parenthesis are used correctly.

$y = x + 7$ $y = x^2 + 3x - 8$

Find intersection(s):

$x + 7 = x^2 + 3x - 8$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3)$ $x = -5$ & $3$

not in the interval

$$\left| \int_1^3 (x^2 + 2x - 15)\,dx \right| + \left| \int_3^6 (x^2 + 2x - 15)\,dx \right|$$

$$\left| \left( \frac{x^3}{3} + x^2 - 15x \right) \Big|_1^3 \right| + \left| \left( \frac{x^3}{3} + x^2 - 15x \right) \Big|_3^6 \right|$$

$$\left| \left(3^2 + 3^2 - 15(3)\right) - \left(\frac{1}{3} + 1 - 15\right) \right| + \left| \left(\frac{6^3}{3} + 6^2 - (5)(6)\right) - \left(3^2 + 3^2 - 15(3)\right) \right|$$

15

6. People have been concerned that, without government aid the price of milk could reach \$6.00 a gallon. A recent study of milk prices show that the price of milk since the beginning of the study can be expressed as

$P(t) = \$2.30 + \$0.30\sqrt{t}$ t is the number of weeks since the beginning of the study.

Find the average price of ~~gas~~ milk from week 4 to week 9 to the nearest penny.

$$\frac{1}{9-4}\int_4^9 (2.30 + .3\sqrt{t})\,dt = \frac{1}{5}\left(2.3t + \frac{(.3)\sqrt{t^3}}{3/2}\right)\Bigg|_4^9$$

$$= \frac{1}{5}\left(2.3(9) + .2(\sqrt{9})^3 - 2.3(4) - .2(\sqrt{4})^3\right)$$

$$= \frac{1}{5}\left(2.3(5) + (.2)(27-8)\right) = 2.3 + \frac{3.8}{5} = \$3.06$$

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7. The marginal profit from the sale of television sets is 112 - .03q where q is the quantity of television sets sold. Find the increase in profit that results from increasing the sales from 100 sets to 200 sets.

$$\int_{100}^{200} (112 - .03q)\,dq = \left(112q - \frac{.03q^2}{2}\right)\Bigg|_{100}^{200}$$

$$112(200-100) - \frac{.03\,\overbrace{(200^2 - 100^2)}^{30,000}}{2}$$

$$11200 - 450 = \$10,750$$

Math 220 – Business Calculus
Spring 2013
Test 4 (140 points)

Name Answer Key
Version B

20 1. A specialty grocery store can sell 300 quarts of pure maple syrup at \$19.00 per quart. For each drop in price of \$0.10 two more quarts can be sold. What price should the store charge for the maple syrup to maximize revenue?

$p = 19.00 - .1x \qquad q = 300 + 2x \qquad R = (19 - .1x)(300 + 2x)$

$R = 5700 - 30x + 38x - .2x^2$

$x$ = # of ~~drops~~ changes in price

$R' = 8 - .4x = 0 \qquad x = 20$

$p = 19 - .1(20) = \$17.00$

2. The quantity of a product that can be sold varies with the price according to this demand function: $q = 2250 - .3p^2$

5 a) What is the general formula for elasticity? $E(p) = -\frac{p}{q}\frac{dq}{dp}$

5 b) What is the equation for elasticity as a function of price for this problem?

$$E(p) = \frac{-p}{2250 - .3p^2}(-.6p) = \frac{.6p^2}{2250 - .3p^2}$$

7 c) Use elasticity to find the price that will yield the highest revenue. No other solution will be accepted.

$$\frac{.6p^2}{2250 - .3p^2} = 1$$

$.6p^2 = 2250 - .3p^2$

$.9p^2 = 2250$

$p^2 = \frac{2250}{.9} = 2500$

$p = \$50$

3 f) At a price of \$45 is the demand elastic or inelastic?

inelastic

3. Evaluate each integral.

Definite integrals should have a numerical expression for an answer. The expression does not need to be simplified except for part c. For example: $(3^2/2) - (1^2/2)$ is a sufficient answer.

Indefinite integrals should satisfy boundary conditions if any are given.

6 a) $\int \left(4x^3 + 3\sqrt[4]{x} + \frac{5}{x^3} + \frac{6+x}{\sqrt{x}}\right) dx$ $= x^4 + \frac{3x^{5/4}}{5/4} + \frac{5x^{-2}}{-2} + \frac{6x^{1/2}}{1/2} + \frac{x^{3/2}}{3/2} + C$

3 b) $\int_{-3}^{4} 4e^{5x}\, dx$ $= \left.\frac{4e^{5x}}{5}\right|_{-3}^{4} = \frac{4}{5}\left(e^{20} - e^{-15}\right)$

5 c) $\int_{1}^{e^5} \frac{4}{x} dx$ Note: you must simplify this problem down to a single number.

$= 4 \ln|x| \Big|_1^{e^5} = 4\left(\ln e^5 - \ln 1\right) = 20$

6 d) $\int_{2}^{5} 8x^2 \ln x\, dx$ $= \left.\frac{8x^3}{3}\ln x\right|_2^5 - \int_2^5 \frac{8x^3}{3}\cdot\frac{1}{x}\, dx$

$dv$ $u$

$v = \frac{8x^3}{3}$ $du = \frac{1}{x}\, dx$

$= \left(\frac{8x^3}{3}\ln x - \frac{8x^3}{9}\right)\Big|_2^5$

$= \left(\frac{8}{3}(5)^3 \ln 5 - \frac{8(5)^3}{9}\right) - \left(\frac{8(2)^3}{3}\ln 2 - \frac{8(2)^3}{9}\right)$

3. continued

6 e) $\int_0^2 (8x^3+2)/\sqrt{3x^4+3x+1}dx$ $= \frac{2}{3}\int_1^{55} \frac{1}{\sqrt{u}}\,du = \frac{2}{3}\left.\frac{u^{1/2}}{1/2}\right|_1^{55}$

$u = 3x^4+3x+1$

$\frac{du}{dx} = 12x^3+3$ $= \frac{4}{3}\left(\sqrt{55}-\sqrt{1}\right)$

$dx = \frac{du}{12x^3+3}$ $3(2)^4+3(2)+1 = 55$

4 f) $\int 2x\sqrt{5+x}dx$ $= \int 2(u-5)\sqrt{u}\,du = \int\left(2u^{3/2}-10u^{1/2}\right)du$

$u = 5+x$ $x = u-5$

$du = dx$

$= \left(\frac{2u^{5/2}}{5/2} - \frac{10u^{3/2}}{3/2} + C\right) = \frac{2(5+x)^{3/2}}{5/2} - \frac{10(5+x)^{3/2}}{3/2} + C$

20 4. Someone is standing on a cliff 300 m above the valley below. That person throws a rock downward with an initial velocity of 30 m sec. The acceleration due to gravity is 9.8 m/sec$^2$. (Remember that downward velocity and acceleration are both negative.) How far above the ground will the rock be after 2 seconds?

$v(t) = \int -9.8\,dt = -9.8t + C$ $v(0) = -30 \Rightarrow C = -30$

$P(t) = \int(-9.8t - 30)\,dt = -4.9t^2 - 30t + C$

$P(0) = 300 \Rightarrow C = 300$

$P(2) = -4.9(2)^2 - 30(2) + 300$

$= -19.6 - 60 + 300 = 220.4$ m

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5. Find the area between these two functions when $1 \leq x \leq 4$: Note: Once you have a numerical answer DO NOT SIMPLIFY! Be sure parenthesis are used correctly.

$y = x + 9 \qquad y = x^2 + 5x - 3$

Find the intersection(s)

$$x + 9 = x^2 + 5x - 3$$
$$x^2 + 4x - 12 = 0$$
$$(x + 6)(x - 2) = 0$$
$$x = -6 \text{ or } 2$$

Not in the interval

$$\left| \int_1^2 (x^2 + 4x - 12)\, dx \right| + \left| \int_2^4 (x^2 + 4x - 12)\, dx \right|$$

$$= \left| \frac{x^3}{3} - 2(x^2) - 12x \Big|_1^2 \right| + \left| \frac{x^3}{3} - 2x^2 - 12x \Big|_2^4 \right|$$

$$= \left| \left( \frac{2^3}{3} - 2(2^2) - 12(2) \right) - \left( \frac{1}{3} - 2 - 12 \right) \right| +$$

$$\left| \left( \frac{4^3}{3} - 2(4)^2 - 12(4) \right) - \left( \frac{2^3}{3} - 2(2)^2 - 12(2) \right) \right|$$

15

6. People have been concerned that, without government aid the price of milk could reach \$6.00 a gallon. A recent study of milk prices show that the price of milk since the beginning of the study can be expressed as

$P(t) = \$2.50 + \$0.60\sqrt{t}$ t is the number of weeks since the beginning of the study.

Find the average price of milk ~~gas~~ from week 4 to week 9 to the nearest penny.

$$\frac{1}{9-4}\int_4^9 (2.5 + .6\sqrt{t})\,dt = \frac{1}{5}\left(2.5t + \frac{.6(\sqrt{t})^3}{3/2}\right)\Big|_4^9\Bigg)$$

$$= \frac{1}{5}\left(2.5(9-4) + .4\left((\sqrt{9})^3 - \sqrt{4}^3\right)\right)$$

$$= 2.5 + \frac{.4(\overbrace{27-8}^{19})}{5} = 2.5 + \frac{7.6}{5} = \$4.02$$

15

7. The marginal profit from the sale of television sets is 123 - .05q where q is the quantity of television sets sold. Find the increase in profit that results from increasing the sales from 100 sets to 200 sets.

$$\int_{100}^{200} (123 - .05q)\,dq = \left(123q - .05\frac{q^2}{2}\right)\Big|_{100}^{200}$$

$$= 123(200-100) + \frac{.05}{2}(\overbrace{200^2 - 100^2}^{30,000})$$

$$= 12300 - 750 = \$11,550$$